# The Network Effects of Fiscal Adjustments* 

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#### Abstract

We study the effects of fiscal consolidations in the United States and their propagation in the production network. We use a narrative approach to identify fiscal adjustments which are exogenous to output fluctuations. Then we apply spatial econometric techniques to separate the total effect of fiscal adjustments into a direct and network component. We find that fiscal adjustments based on increased taxation are more recessionary than those based on spending cuts. Moreover, one quarter of the difference in their total output effect is explained by the stronger network propagation of taxes relative to government spending.


Keywords: industrial networks, fiscal adjustment plans, output growth, applied spatial econometrics.
JEL codes : E60, E62.

[^0]
## 1 Introduction

Economic theory and good practice suggest that a government should run a deficit during recessions, when tax revenues are low and government spending is high due to fiscal stabilizers like unemployment subsidies. The same holds during periods of temporarily high spending needs, when a government must cope with catastrophes such as financial crises, natural disasters, or wars. These deficits should be balanced by surpluses during economic booms and when spending needs are low. ${ }^{1}$ As global economies recover from the COVID19 crisis and return to growth, fiscal consolidations play a crucial role in balancing government budgets and bringing sovereign debt below the "maximum sustainable debt" threshold (see e.g., Collard, Habib, and Rochet (2015)). ${ }^{2}$ In this context, understanding the transmission mechanism and output effects of fiscal consolidations is crucial for policymakers hoping to design optimal fiscal adjustment plans. ${ }^{3}$

As recently reported in Ramey (2019), the fiscal policy literature has consistently found a new empirical fact: fiscal consolidations due to higher taxes imply larger output losses compared to consolidations due to reductions in government spending. This pattern has been confirmed in recent studies which compare the effects of austerity measures across a panel of countries (see e.g., Guajardo, Leigh, and Pescatori (2014) and Alesina, Favero, and Giavazzi (2015)). However, there is little work investigating the underlying reasons for such an asymmetric response.

In this paper, we explore production networks as a potential explanation for the asymmetric output response of fiscal consolidations. Contrary to existing literature, we restrict our analysis to one single country, namely, the United States. This has two main benefits. Firstly, we are able to estimate effects specific to the US and thus provide more reliable guidance from a policymaking perspective, as multi-country analysis tends to report country-average effects. Secondly, we are able to exploit rich industry-level data to track the effects of fiscal consolidations at a more disaggregated level. As a result, we shed light on the transmission mechanism of fiscal policy and quantify its

[^1]propagation through the industrial network.
In particular, we motivate our work with the following questions. What are the effects of fiscal adjustments in the United States? Are tax-based fiscal consolidations more recessionary than expenditure-based consolidations, as highlighted by the country-panel analysis? Can asymmetries in the inputoutput network explain this difference? To provide an answer, we study the effects of fiscal consolidations implemented in the Unites States from 1978 to 2014. These effects propagate through a 62 -industry production network.

Regarding the first two questions, we find that tax-based (TB henceforth) fiscal adjustments have a recessionary output multiplier over two years of $1.4 \%$ while the effects of expenditure-based (EB henceforth) fiscal plans are not statistically different from zero. These results are in line with those obtained by the current state of the literature which uses a panel of OECD countries. Moreover, we answer the third question using spatial econometric techniques, assuming that the observed units, 62 industries, are "spatially connected" via an input-output production network. The spatial framework allows us to decompose the aggregate total effects of fiscal consolidations into a direct and a network effect. The former represents the direct impact of the fiscal shock on each industry while the latter represents the spillover effects from other industries hit by the same aggregate shock. In turn, we are able to investigate if the stronger recessionary effects of TB fiscal consolidations relative to EB are explained by differences in the network propagation mechanism of these shocks.

Our baseline results suggest that $27 \%$ of the total effect of TB fiscal consolidations come from network spillovers. On the other hand, network effects of EB plans are more modest and less robust, with only $11 \%$ of the total output effect coming from the network. Overall, the stronger network effect of TB plans explains close to one-fourth of the differences in the total effects of TB and EB plans. Networks thus provide a partial explanation of asymmetry in the output response of these two types of fiscal consolidations.
In addition to these results, this paper has two other original contributions. To the best of our knowledge, we are the first to study and detect input-output spillovers of taxes. We find that a few key suppliers in the economy are responsible for most of the network propagation of tax shocks. ${ }^{4}$ This result is consistent with Ozdagli and Weber (2017), who study upstream propagation of monetary policy shocks. As noted in Ozdagli and Weber (2017), spatial

[^2]models of the macro-economy are a useful tool for understanding the sources and transmission mechanism of aggregate shocks. As far as we know, recently there is a growing literature, which makes use of this class of models in macro including but not limited to Ozdagli and Weber (2017), and Di Giovanni and Hale (2021).

## Related Literature

First of all, our paper relates to the literature of fiscal consolidations: Guajardo, Leigh, and Pescatori (2014), Alesina, Favero, and Giavazzi (2015) and Alesina, Barbiero, et al. (2017). Unlike these papers, we consider a panel of US industries rather than countries, and we are the first to study the network effects of fiscal consolidations.
Alesina, Barbiero, et al. (2017) also propose a theoretical explanation for the stronger effect of TB fiscal consolidations. They introduce the possibility of persistent adjustment plans in a standard New Keynesian framework to show that when fiscal adjustments are close to permanent, spending cuts are less recessionary than tax hikes. Karamysheva (2022), using the VARX model, offers another explanation of more recessionary effects of TB plans, based on financial market and uncertainty channels. Brinca et al. (2021) provide evidence both theoretically and empirically that income inequality plays an important role in explaining the transmission mechanism of fiscal consolidation. On the other hand, we provide an alternative, network-based explanation of the asymmetric output effects of TB and EB plans.
Secondly, our work relates to the seminal works of Gabaix (2011), Acemoglu, Carvalho, et al. (2012), which develop the role of production networks in amplifying the effects of localized shocks. ${ }^{5}$ However, unlike these papers, our work adopts a spatial framework to determine the extent to which the total effects of fiscal policy can be attributed to network transmission. This point has been highlighted in Ozdagli and Weber (2017), who perform a similar analysis to study the propagation of monetary policy shocks in the US stock market.
Acemoglu, Akcigit, and Kerr (2016) study the asymmetric propagation in the production network of demand and supply shocks. In particular, they find that government spending shocks uniquely propagate upstream in the production network, from customers to suppliers. Bouakez, Rachedi, and Santoro (2020a)

[^3]find that the sectors that react the most to government spending shocks are those located upstream in the production network, and Bouakez, Rachedi, and Santoro (2020b) show that the aggregate multiplier is relatively larger when government spending is tilted towards downstream industries. Unlike them, we study empirically the propagation of a special type of fiscal shock, namely TB and EB fiscal consolidations.
Thirdly, this work relates to the literature on fiscal policy at an industry level: Ramey and Shapiro (1999), Perotti (2007) and Nekarda and Ramey (2011). In particular, Nekarda and Ramey (2011) focus on government purchases in manufacturing industries and find evidence in support of the Neo-Classical model. They also construct a comprehensive measure of government purchases which takes into account downstream linkages. Building on this work, we provide analysis of the transmission mechanism of fiscal policy at an industry level. Additionally, we enrich this analysis by using all the industries in the economy and by integrating them into a production network.
Cox et al. (2020) study public procurement contracts and find large sectoral bias in government spending. Our industry analysis thus takes into account the sectoral heterogeneity of fiscal policy effects. Auerbach, Gorodnichenko, and Murphy (2019) use city-level data on local defense public procurement and find large fiscal (first-order) spillovers among industries. Their results contradict our finding of weak propagation of EB plans. However, it is hard to provide a direct comparison between our two results since we use different levels of aggregation and we study the effects of fiscal consolidations.

The rest of the paper is organized as follows. In Section 2, we illustrate how fiscal adjustment plans identify exogenous fiscal consolidation policies. This section also studies the aggregate effects of fiscal consolidations and provide a theoretical rationalization of the underlying transmission mechanism. Section 3 illustrates our results. Section 4 provides some robustness checks and Section 5 concludes.

## 2 Fiscal Adjustments Plans in the US

Measuring the propagation of fiscal adjustments requires the identification of an exogenous demand and supply shocks. Our identification strategy thus relies on the narrative analysis of fiscal adjustment plans. This strategy is a recent innovation in the fiscal policy literature and employs narrative exogenous shocks as a proxy for fiscal consolidation policies. This strategy was introduced in Alesina, Favero, and Giavazzi (2015) to take into account the
fact that fiscal adjustments are implemented through multi-year plans with both an intertemporal and an intratemporal dimension.
The intratemporal dimension refers to the fact that fiscal consolidations are implemented with a mix of tax increases and spending cuts. Tax and the expenditure components of the adjustments are correlated since governments decide first on the size of the adjustment, and then on its composition in terms of expenditures and revenues. The intertemporal dimension is important since fiscal adjustments are implemented via multi-year plans with measures upon announcement (the unanticipated component of the plan) and measures announced for subsequent years (the anticipated component of the plan). In particular, each country has a specific "recipe" to implement fiscal consolidations: some countries prefer to unexpectedly raise taxes without cutting expenditures, while others announce large future cuts in spending and only marginally increase taxes. Alesina, Favero, and Giavazzi (2015) refer to this as the country-specific "style of the plan".
These complications make identifying pure and isolated tax hikes and spending cuts during years of fiscal consolidation a difficult, if not impossible, task. Fiscal plans provide an effective tool to circumvent these difficulties when studying austerity policies.

### 2.1 Modeling Fiscal Plans:

From a mathematical standpoint, plans are sequences of fiscal corrections, announced at time $t$ and implemented between $t$ and $t+K$, where $K$ is the anticipation horizon. In each year $t$, two types of fiscal corrections are possible:

1. The unanticipated fiscal shock, that is, the surprise change in the primary surplus at time $t$, which we denote by:

$$
f_{t}^{u}:=t a x_{t}^{u}+e x p_{t}^{u},
$$

where $t a x_{t}^{u}$ is the surprise increase in taxes announced and implemented at time $t$, while $\exp _{t}^{u}$ is the surprise reduction in government expenditure also announced and implemented at time $t$.
2. The anticipated fiscal shock: the change in the primary surplus at time $t$, which had already been announced in the previous years and is either implemented in year $t$ or scheduled to happen within $K$ years. In particular, we denote as $t a x_{t, j}^{a}$ and $e x p_{t, j}^{a}$ the tax and expenditure changes announced by the fiscal authorities at date $t$ with an anticipation horizon of $j$ years (i.e., to be implemented in year $t+j$ ). Therefore, we further distinguish between:
(a) The anticipated implemented shock: scheduled in the past and implemented in year $t$ :

$$
f_{t}^{a}:=\operatorname{tax}_{t, 0}^{a}+\exp _{t, 0}^{a}
$$

(b) The anticipated future shocks: sum of scheduled tax and government spending changes which have to be implemented within $K$ years from their announcement:

$$
f_{t}^{f}:=\sum_{j=1}^{K} t a x_{t, j}^{a}+\sum_{j=1}^{K} \exp _{t, j}^{a} .
$$

In a fiscal adjustment database, as long as no policy revision takes place, the anticipated shocks roll over year-by-year. In formulae:

$$
\text { tax } x_{t, j}^{a}=\underbrace{t a x_{t-1, j+1}^{a}}_{\text {Old shock, rolled over }} \quad \exp _{t, j}^{a}=\underbrace{e x p_{t-1, j+1}^{a}}_{\text {Old shock, rolled over }} .
$$

However, if from one year to another, a policy revision takes place, then, the new anticipated future shock will embed such change: ${ }^{6}$

$$
\begin{array}{ll}
t a x_{t, j}^{a}=\underbrace{t a x_{t-1, j+1}^{a}}_{\text {Old shock, rolled over }}+\underbrace{\left(t a x_{t, j}^{a}-t a x_{t-1, j+1}^{a}\right)}_{\text {Policy Revision }}, & \text { with } j \geq 1 \\
e x p_{t, j}^{a}=\underbrace{e x p_{t-1, j+1}^{a}}_{\text {Old shock, rolled over }}+\underbrace{\left(e x p_{t, j}^{a}-e x p_{t-1, j+1}^{a}\right)}_{\text {Policy Revision }}, & \text { with } j \geq 1
\end{array}
$$

We adopt the annual database on fiscal adjustment plans constructed by Alesina, Favero, and Giavazzi (2015) and consider only fiscal consolidations that occurred in the US from 1978 to 2014. They identify fiscal adjustments exogenous with respect to output fluctuations using a narrative identification method. This approach is similar to C. D. Romer and D. H. Romer (2010), who identify exogenous tax shocks from presidential speeches, congressional debates, budget documents, and congressional reports. From these documents, they identify the size, timing, and principal motivation for all major postwar tax policy actions. Legislated changes are then classified into two categories:

[^4]1) endogenous, if induced by short-run counter-cyclical concerns; 2) exogenous, if taken in response to the state of government debt (deficit-driven). ${ }^{7}$ As mentioned earlier, fiscal adjustment plans allow us to control for the intertemporal and intratemporal correlation, which we report in Table I:

Table I: Inter- and Intra-temporal correlation matrix of Fiscal Adjustments Plans in the US

|  | $\operatorname{tax}_{t}^{u}$ | $\operatorname{tax}_{t, 0}^{a}$ | $\operatorname{tax}_{t}^{f}$ | $\exp _{t}^{u}$ | $\exp _{t, 0}^{a}$ | $\exp _{t}^{f}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{tax}_{t}^{u}$ | 1 | 0.041 | 0.570 | 0.596 | -0.126 | 0.105 |
| $\operatorname{tax}_{t, 0}^{a}$ |  | 1 | 0.038 | 0.098 | 0.361 | 0.310 |
| $\operatorname{tax}_{t}^{f}$ |  |  | 1 | -0.047 | 0.019 | 0.180 |
| $\exp _{t}^{u}$ |  |  |  | 1 | -0.050 | 0.014 |
| $\exp _{t, 0}^{a}$ |  |  |  |  | 1 | 0.782 |
| $\exp _{t}^{f}$ |  |  |  |  |  | 1 |

Table I: linear correlation matrix of legislated changes in taxes and expenditure identified by the narrative analysis. Sample: annual data from 1978 to 2014 of US fiscal adjustment plans from Alesina, Favero, and Giavazzi (2015). In blue is reported the intra-temporal correlation (between each component of taxes and expenditures. In green is the inter-temporal correlation (within tax or expenditure component, but between components with different timing). In black we have a mix of the two: correlation between tax and expenditure components with different timing.

Notice from Table I that the intra-temporal correlation between unanticipated tax and unanticipated expenditure adjustments is $60 \%$ (blue figures in Table I). Similarly, the (inter-temporal) correlation between future and anticipated components of expenditure is $78 \%$ (green figures in Table I). As both the inter-temporal and the intra-temporal dimension matter, it is worth considering multi-year fiscal plans instead of individual measures of tax and government spending shocks.
Since this source of correlation confounds the effects of taxes and expenditures, we need to classify plans into mutually exclusive categories which can be simulated independently. We can then take into account the inter-temporal correlation within each category. To this end, we exploit the fact that not all the plans are the same. Some fiscal plans are designed to increase taxes more than cut expenditures and are labeled as TB (tax-based). On the contrary, those plans which rely more on expenditure cuts rather than tax hikes are

[^5]labeled as EB (expenditure-based).
For instance, the criterion which determines whether a fiscal consolidation is labeled as TB can be written:
\[

$$
\begin{equation*}
\underbrace{\left(t a x_{t}^{u}+t a x_{t, 0}^{a}+\sum_{j=1}^{K} t a x_{t, j}^{a}\right)}_{\text {overall tax hike in } t}>\underbrace{\left(e x p_{t}^{u}+e x p_{t, 0}^{a}+\sum_{j=1}^{K} e x p_{t, j}^{a}\right)}_{\text {overall expenditure cut in } t} . \tag{1}
\end{equation*}
$$

\]

Criterion (1) is saying that if the overall tax hike in year $t$ exceeds the overall spending cut, then we label year $t$ as a year of TB fiscal consolidation. We keep track of these years by constructing two dummy variables, $T B_{t}$ and $E B_{t}$, which are equal to one if year $t$ is labeled as TB or EB , respectively. By construction, TB and EB plans are mutually exclusive. That is, EB and TB plans cannot occur simultaneously. This lets us simulate separately the effect of TB and EB plans while preserving, within each type of plan, the observed intra-temporal correlation between adjustments on government's revenues and expenditure.
Figure 1 plots our fiscal adjustment plans database. This contains all of the nominal changes in taxes and expenditure, scaled by GDP of the year before the consolidation occurs to avoid potential endogeneity issues. Moreover, the future component of the fiscal adjustment plan has a maximum anticipation horizon of three years $(K)$. This is in line with the small numbers of occurrences of policy shifts anticipated four and five years ahead, and is consistent with the database in Pescatori et al. (2011). The top row of Figure 1 illustrates the three components of fiscal adjustments interacted with the dummy $T B_{t}$ to identify the components of tax-based fiscal consolidations. The bottom row does the same for expenditure-based fiscal consolidations.

We assess the goodness of our orthogonalization criterion (1), by showing in Figure 2 the share of tax increases and spending cuts of each total fiscal adjustment, $f_{t}^{u}+f_{t}^{a}+f_{t}^{f}$.

Figure 2 shows that the labeling of fiscal adjustment into EB or TB plans, by means of criterion (1), is never marginal: i. TB plans are all pure tax hikes except for the year 1988, which is the result of a hybrid fiscal plan with only $30 \%$ in spending cuts; ii. EB plans are mainly made up of spending cuts with only $20 \%$ of policy changes coming from a tax increase, on average. Figure 2 also illustrates the timing of fiscal consolidations in the US: i. there are two periods of TB fiscal adjustments ( $T B_{t}=1$ ) between 1978-1981 and 1985-1988; ii. there are three periods of EB fiscal adjustments $\left(E B_{t}=1\right)$ between 19901992, 1993-1998 and 2011-2013.

Figure 1: Fiscal Adjustments Plans - United States 1978-2014


Figure 2: Fiscal Adjustment Composition



To summarize, we classify fiscal consolidations into TB and EB fiscal ad-
justment plans to account for the observed correlation between tax and expenditure adjustments. This correlation comes from the fact that policy makers implement fiscal consolidations by adopting multi-year fiscal plans with both tax hikes and spending cuts.

Finally, we highlight that fiscal consolidations censor changes in G and T above and below 0 , respectively, by construction. ${ }^{8}$ Therefore, estimates of their economic effects are valid for this type of fiscal policy only. Simply put, we do not estimate tax and government spending multipliers. ${ }^{9}$ However, if the United States plans to undertake either a TB or an EB fiscal consolidation, our estimates are externally valid and can be used as a benchmark for policymakers.

### 2.2 Aggregate Effects of Fiscal Consolidations in the US

The first step of our analysis is to study the aggregate effects of fiscal consolidations in the US. We estimate the impulse response functions of EB and TB plans using a truncated moving average (MA) representation as in C. D. Romer and D. H. Romer (2010) but where the shocks are given by the fiscal consolidations as in Alesina, Favero, and Giavazzi (2015). So we simulate the response to an unanticipated component taking into account that it is accompanied by the announcement of future changes. Following Alesina, Favero, and Giavazzi (2015) we compute impulse response functions as a difference between the forecast obtained conditionally on a fiscal adjustment plan and the forecast with no plan. ${ }^{10}$ Figure 3 shows the estimated cumulative impulse response function of output and employment growth rates using quarterly data from 1978Q1 to 2014Q4.

The left panel of Figure 3 shows that TB plans trigger a cumulative drop of output and employment by $4 \%$ and $2 \%$ respectively. On the contrary, EB plans do not seem to be recessionary. This result is in line with the findings of the fiscal policy literature.

We repeat the analysis on each component of GDP and report the estimated cumulative impulse response functions in Figure 4.

[^6]Figure 3: Output and Employment Response to Austerity


We find that TB fiscal plans are associated with lower than average consumption growth while the other components of GDP do not respond. On the contrary, EB fiscal consolidations exhibit increases in each component of private GDP which are not statistically significant while government spending falls significantly.

Having illustrated what happens to all components of output we turn our attention on the type of fiscal policy change implemented during years of austerity. Firstly, we estimate the effects of fiscal plans on the growth rates of government receipts shares of output using again a truncated moving average. Figure 5 shows the cumulative response of government receipts shares of output coming from excise/production taxes and payroll taxes. Other types of government receipts such as corporate tax, income tax, estate/gift tax and custom duties are not affected (see Appendix A).

Figure 4: GDP by Austerity


Variables are in real dollars (source NIPA). The darker region refers to the 68\% confidence level while the lighter region represents the $95 \%$ confidence level obtained via block-bootstrap. Quarterly data. Sample goes from $1978 Q 1$ to $2014 Q 4$.

Looking at the top-left panel, we find that excise/production taxes are the main component of government receipt affected by TB fiscal consolidations,

Figure 5: Government Revenues Affected by Austerity

increasing its share of GDP by $12 \%$ over a three years horizon. On the contrary, the change in excise taxes share of output during EB fiscal plans is not statistically different from zero (see top-right panel). Looking at the bottom panels, government receipts share of output coming from payroll taxes appear to increase during both TB and EB fiscal consolidations. This is not surprising if we look back at Figure 2: EB plans also have a small fraction of tax increases in their style (i.e. intra-temporal correlation).

Secondly, we study what happens to government expenditures during years of fiscal consolidations. In particular, we break down government spending, G , into two components: procurement spending and the residual part of G , non-procurement spending. ${ }^{11}$ Figure 6 shows the cumulative impulse response function of the two components of G as share of GDP. Notice from the right

[^7]column that only EB plans affect government expenditures. In fact, recall from Figure 2 that TB plans are pure tax hikes.

Figure 6: Government Expenditures Affected by Austerity


Cumulative IRFs of government expenditure two components shares of GDP. The darker region refers to the 68\% confidence level while the lighter region represents the 95\% confidence level, obtained via residual block-bootstrap as suggested in Jentsch and Lunsford (2019a). Quarterly data. Sample goes from 1978Q1 to 2014 Q4.

Overall, the aggregate results show that tax-based austerity plans (i) were recessionary, (ii) hit especially consumption and (iii) were implemented by increasing payroll and excise taxes. The expenditure side was unaffected. On the contrary, spending-cuts austerity was characterized by mild and statistically insignificant increases in output and was mainly done via equal cuts in procurement spending and the rest of government consumption expenditure. ${ }^{12}$

[^8]
### 2.3 Fiscal Plans and Production Networks

In the previous section we highlight an important difference between TB and EB plans: positive changes in excise/production taxes are unique to TB plans while procurement spending cuts are unique to EB fiscal consolidations. Notice that reducing procurement spending and increasing excise/production have completely different transmission mechanisms. For instance, imagine a $n$-sectors static Cobb-Douglas economy as in Acemoglu, Akcigit, and Kerr (2016). In their model a change in government purchases behaves as a demand shock which propagates upstream in a production network: an industry affected by a demand shock propagates the shock to all its suppliers of input. On the contrary, excise/production taxes behave as supply shocks which propagate downstream: an industry affected by a supply shock passes the shock to all its customers. Step by step, the shock trickles down to consumers. Notice that this asymmetric transmission mechanism of taxes and government purchases is consistent with the asymmetric response of consumption during years of TB and EB fiscal consolidations.

In this section we explore the theoretical propagation of these types of fiscal policy through the lens of a simple static model with production network, which is a slightly modified version of Acemoglu, Akcigit, and Kerr (2016). ${ }^{13}$

In this model the economy is inhabited by a representative agent with Cobb-Douglas utility over $n$-goods. On the production side, the representative sector $i$ maximizes profits:

$$
\max _{l_{i},\left\{x_{i j}\right\}_{j=1}^{n}}(1-\tau) \cdot p_{i} \cdot \underbrace{\left(l_{i}^{\alpha_{i}^{l}} \cdot\left(\prod_{j=1}^{n} x_{i j}^{a_{i j}}\right)^{\rho}\right)}_{:=y_{i}}-w l_{i}-\sum_{j=1}^{n} p_{j} x_{i j}
$$

where $\tau$ is the excise/production tax, $p_{i}$ is the price of output $i, l_{i}$ is the labor input of sector $i, x_{i, j}$ is the quantity of intermediate good $j$ purchased by sector $i$ as input of production, $w$ is the wage and $y_{i}$ is output of producer of good $i .^{14}$ The resource constraint of the economy is:

$$
y_{i}=c_{i}+\sum_{j=1}^{n} x_{j i}+G_{i}
$$

where $c_{i}$ and $G_{i}$ are consumption and government purchases of good $i$ respectively, while $x_{j i}$ is the quantity of good $i$ used as input of production by sector

[^9]$j$.

## EB Plans

In this static economy a change in government purchases has the following output effect: ${ }^{15}$

$$
\begin{equation*}
d \log y_{i}=\rho \cdot \sum_{j=1}^{n} \underbrace{a_{j i} \cdot \frac{p_{j} \cdot y_{j}}{p_{i} \cdot y_{i}}}_{:=\hat{a}_{j i}} \cdot d \log y_{j}+\frac{G_{i}}{y_{i}} \cdot d \log G_{i} \tag{2}
\end{equation*}
$$

which in matrix form becomes:

$$
d \log _{n \times 1} \mathbf{y}=\rho \cdot \hat{A}^{T} \cdot d \log \mathbf{y}+\Lambda \cdot d \log \boldsymbol{G}
$$

where $\Lambda=\operatorname{diag}\left(G_{1} / y_{1}, . ., G_{n} / y_{n}\right)$ and $\hat{A}=\left[a_{j i} \cdot\left(p_{j} \cdot y_{j}\right) /\left(p_{i} \cdot y_{i}\right)\right]_{i, j=1, \ldots, n}$. Moreover, in equilibrium, we also have:

$$
\underset{n \times n}{\hat{A}^{T}} \propto\left[\frac{p_{i} \cdot x_{j i}}{p_{i} \cdot y_{i}}\right]_{i, j=1, \ldots, n}=\left[\frac{\operatorname{SALES}_{i \rightarrow j}}{\mathrm{OUTPUT}_{i}}\right]_{i, j=1, \ldots, n}
$$

The $i-j$ element of $\hat{A}^{T}$ is proportional to the sales of sector $i$ to sector $j$, relative to its output, $y_{i}$. Therefore, the transmission of government purchases works from customers (sector $j$ ) to suppliers (sector $i$ ). Finally, to understand the transmission mechanism of government purchases, it is convenient to solve the above expression and then expand it using the definition of geometric sum:

$$
\begin{align*}
d \log \mathbf{y} & =\left(I_{n}-\rho \cdot \hat{A}^{T}\right)^{-1} \cdot \Lambda \cdot d \log \boldsymbol{G} \\
& =\left(I_{n}+\rho \cdot \hat{A}^{T}+\rho^{2} \cdot\left(\hat{A}^{T}\right)^{2}+\ldots\right) \cdot \Lambda \cdot d \log \boldsymbol{G} \tag{3}
\end{align*}
$$

Equation (3) is saying that spending cuts propagate upstream in the production network. For example, consider a spending cut on good $j$ (i.e. $d \log G_{j}<$ $0)$. Firstly, output is directly reduced and this first order effect is represented by matrix $I_{n}$ in the geometric sum expansion. Secondly, sector $j$ reduces the amount of input it needs. Therefore, for each sector $i$, supplier of $j$, we have that $x_{j i}=$ SALES $_{i \rightarrow j}$ decreases. This is a second order effect working via $\rho \cdot \hat{A}^{T}$. Thirdly, suppliers of suppliers of producer of good $j$ also face an indirect effect and so on and so forth. Since the propagation of the spending cut happens from customers to suppliers, we refer to this type of transmission mechanism as upstream propagation.

[^10]Example: 3 Sectors Economy. We further clarify this type of propagation using a simple numerical example illustrated in Figure 7. In the example we

Figure 7: Example of Spending Cut


Figure 7: vertically integrated 3 sectors economy. $\hat{a}_{21}=0.5$ means that sector 1 sells $50 \%$ of its output to sector 2. $\hat{a}_{32}=0.9$ means that sector 2 sells $90 \%$ of its output to sector 3. $\hat{a}_{31}^{2}:=\hat{a}_{21} \cdot \hat{a}_{32}=0.45$, it means that $45 \%$ of sector's 1 output is indirectly purchased by sector 3 via sector 2.
have an economy with three sectors which are vertically integrated: sector 1 supplies sector 2 which supplies sector 3 . The upstream input-output matrix is given by $\hat{A}^{T}$, which is sparse everywhere but in positions $1-2$ and 2-3, which reflect the fact that sector 1 supplies sector 2 and sector 2 supplies sector 3 . Moreover, $\left(\hat{A}^{T}\right)^{2}$ represents the second order connection, that is, the suppliers of the suppliers. This production network is characterized by a single second order connection: sector 1 indirectly supplies sector 3 via sector 2 . In fact, $\left(\hat{A}^{T}\right)^{2}$ is sparse everywhere but in position 1-3.

Suppose the government cuts by 0.1 the demand from sector 2 . Suppose also that government spending shares of sectoral output are all the same before the policy change, then, the output effect implied by Equation (3) is:
$d \log \boldsymbol{y}=\left(\left[\begin{array}{lll}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right]+\rho\left[\begin{array}{ccc}0 & 0.5 & 0 \\ 0 & 0 & 0.9 \\ 0 & 0 & 0\end{array}\right]+\rho^{2}\left[\begin{array}{ccc}0 & 0 & 0.45 \\ 0 & 0 & 0 \\ 0 & 0 & 0\end{array}\right]\right) \cdot\left[\begin{array}{c}0 \\ -0.1 \\ 0\end{array}\right]=-\left[\begin{array}{c}0.05 \cdot \rho \\ 0.1 \\ 0\end{array}\right]$

Notice that sector 2 is hit directly by the shock and its output shrinks exactly by 0.1. Afterwards, the shock travels upstream in the production network, hitting sector 1 because it is the input-supplier of sector 2 . On the contrary sectors located downstream in the network are not affected. Finally, the aggregate output effect is given by the average of the sectoral output changes: $d \log y=1 / 3 \cdot(0.1+\rho \cdot 0.05)$. Notice that the stronger the intensity of the input-output connections, represented by $\rho$, the stronger the aggregate output effect.

Therefore, the model suggests that during years of EB fiscal consolidations sectors located upstream in the production network should be negatively affected by input-output spillovers coming from those cuts in government purchases which we documented in the previous section. Moreover, the total output effect and the network-effect are proportional to the intensity of the upstream propagation during those years, represented in the model by $\rho$.

## TB Plans

When the government increases the production/excise tax, the model returns the following output change:

$$
\begin{equation*}
d \log y_{i}=\rho \cdot \sum_{j=1}^{n} a_{i j} \cdot d \log y_{j}-\psi_{i} \cdot d \log \tau_{i} \tag{4}
\end{equation*}
$$

where $\psi_{i}>0 .{ }^{16}$ In matrix form the above expression becomes:

$$
d \log _{n \times 1} \mathbf{y}=\rho \cdot A \cdot d \log \mathbf{y}-\Psi \cdot d \log \boldsymbol{\tau}
$$

where $\Psi=\operatorname{diag}\left(\psi_{1}, \ldots, \psi_{n}\right)$ and $A=\left[a_{i j}\right]_{i, j=1, \ldots, n}$. The economic interpretation of $A$ is the opposite of the one of $\hat{A}^{T}$. In fact, in equilibrium we have:

$$
\underset{n \times n}{A} \propto\left[\frac{p_{j} \cdot x_{i j}}{p_{i} \cdot y_{i}}\right]_{i, j=1, \ldots, n}=\left[\frac{\operatorname{SALES}_{j \rightarrow i}}{\mathrm{OUTPUT}_{i}}\right]_{i, j=1, \ldots, n}
$$

that is, the $i-j$ element is proportional to the purchase of good $j$ by sector $i$ relative to its output, $y_{i}$. In this case, the transmission mechanism works from suppliers (sector $j$ ) to customers (sector $i$ ).

Once again, we solve the above expression and then expand it using the definition of geometric sum:

$$
\begin{equation*}
d \log _{n \times 1} \mathbf{y}=-\left(I_{n}+\rho \cdot A+\rho^{2} \cdot A^{2}+\ldots\right) \cdot \Psi \cdot d \log \boldsymbol{\tau} \tag{5}
\end{equation*}
$$

[^11]In this case, the shock propagates in the production network from suppliers to customers via matrix $A$. In particular, the underlying transmission of the shock works through price increases. In fact, in equilibrium we have:

$$
d \log p_{i}=\rho \cdot \sum_{j=1}^{n} a_{i j} \cdot d \log p_{j}+\frac{\tau_{i}}{1-\tau_{i}} \cdot d \log \tau_{i}
$$

When taxes increase, production becomes more costly and prices go up. A price increase impact the direct customers of the taxed producer, which react by increasing their price too. Eventually, the price-increase trickles down to consumers who respond by decreasing consumption:

$$
d \log c_{i}=-d \log p_{i}
$$

Therefore, we refer to this type of transmission mechanism as downstream propagation.

Consider the example of the vertically integrated 3-sectors economy of Figure 7. In this case, a tax specific shock to sector 2 , would have a direct effect on sector 2. Secondly, the shock would travel downstream in the production network via matrix $A$ and would hit sector 3 . The tax increase hits the supplier (sector 2), which increases prices, thus damaging its customer (sector 3). On the contrary, sector 1, located upstream in the supply chain, would not be affected by the tax shock.

Therefore, the model suggests that during years of TB fiscal consolidations, when excise/production taxes were increased, sectors located downstream, as well as consumers, are hit by negative spillovers from sectors located upstream in the production network.

## 3 The Network Effect of Fiscal Plans: Results

In the previous section we illustrated that fiscal policy changes implemented during years of fiscal consolidations, namely excise/production tax increases and procurement spending cuts, propagate downstream and upstream in the production network. In particular, expression (2) suggests that changes in sectoral output, $d \log \boldsymbol{y}$, during years of EB fiscal adjustment plans, should be proportional to an upstream spatial lag, $\rho \cdot \hat{A}^{T} \cdot d \log \boldsymbol{y}$ and the spending shock, that is, the EB fiscal adjustment plan. Similarly, expression (4) suggests that changes in sectoral output during years of TB fiscal adjustment plans, should be proportional to a downstream spatial lag, $\rho \cdot A \cdot d \log \boldsymbol{y}$ and the tax shock, that is, the TB fiscal adjustment plan.

Therefore, the most natural regression equation to test the intensity of the propagation of fiscal consolidations in the production network is: ${ }^{17}$

$$
\left.\begin{array}{rl}
\Delta \log y_{i, t} & =a_{i}+(\rho^{\text {down }} \cdot \underbrace{\sum_{j=1}^{n} a_{i j} \cdot \Delta \log y_{j, t}}_{\Delta y_{i, t}^{\text {down }}}+\psi_{i} \cdot(\underbrace{\tau_{u} \cdot f_{t}^{u}+\tau_{a} \cdot f_{t}^{a}+\tau^{f} \cdot f_{t}^{f}}_{\text {Tax Increases }})) \cdot T B_{t}+ \\
& +(\rho^{u p} \cdot \underbrace{\sum_{j=1}^{n} \hat{a}_{j i} \cdot \Delta \log y_{j, t}}_{\Delta y_{i, t}^{u p}}+\lambda_{i} \cdot(\underbrace{\gamma_{u} \cdot f_{t}^{u}+\gamma_{a} \cdot f_{t}^{a}+\gamma^{f} \cdot f_{t}^{f}}_{\text {Spending Cuts }})) \cdot E B_{t}+\nu_{i, t} \tag{6}
\end{array}\right)
$$

Firstly, Equation (6) includes industry fixed effects, $a_{i}$, industry weights $\psi_{i}$ and $\lambda_{i}$ for TB and EB fiscal consolidations respectively and $\nu_{i, t}$, a serially uncorrelated, heteroskedastic error term. We allow for heteroskedasticity since sectors exhibit different volatility in growth rates in the data. Secondly, the first line of Equation (6) contains the downstream spatial variable which captures the downstream spillovers of the unanticipated, announced and future tax increases. Both are interacted with $T B_{t}$, the dummy variable which is one during years of TB fiscal consolidations. Similarly, the second line of Equation (6) captures the effects of EB fiscal consolidations as well as its upstream spillovers.

Our econometric specification relates to Alesina, Favero, and Giavazzi (2015), who regress country-level output growth on the 3 components of TB and EB country-specific fiscal plans. Unlike them, we focus on a single country, the United States, by breaking down its economy into $n=62$ industries. Furthermore, we enrich their specification with two spatial variables to take into account the input-output connections among sectors and break down the output effect into a direct and a network effects. This is similar to the empirical approach in Acemoglu, Akcigit, and Kerr (2016) and Ozdagli and Weber (2017).

[^12]
### 3.1 Model Estimation

We focus on a partition of the US economy made by 62 industries, observed from 1978 to 2014 at a yearly frequency. Details on the data construction are reported in Appendix B.

We report results based on the static spatial panel autoregressive models specified by Equation (6). The spatial models allow us to track the effect of EB and TB fiscal adjustment plans on industry output growth, while controlling for downstream and upstream spillovers. When estimating the corresponding parameters, standard OLS delivers inconsistent results since the spatial variables are endogenous. We overcome this problem using spatial econometric techniques. In particular, we use a modified version of the Bayesian Markov Chain Monte Carlo (MCMC) illustrated in LeSage and Pace (2009) to estimate the parameters of equation (6). We also report Maximum Likelihood Estimates (MLE) for two main reasons: (i) if all priors are non-informative, then the Bayesian MCMC should exactly return the MLE, (ii) MLE properties of spatial panel autoregressive models with fixed effects are well known (see Yu, DeJong, and Lee (2008)). ${ }^{18}$ The derivation of the Bayesian MCMC and of the MLE as well as other technical details are remanded to Appendix C.3.

Table II reports descriptive statistics of the estimated parameters of interest of model (6):

Firstly, looking at Table II, we notice that the maximum likelihood estimates are very close to the expected value and standard deviation of the posterior distributions estimated by MCMC. This is a consequence of using mainly non-informative priors. Secondly, we notice that during years of TB fiscal consolidations, the downstream spatial correlation is much stronger than the upstream spatial correlation during EB fiscal consolidations. In fact, looking at the quantiles of the posterior distribution of $\rho^{u p}$, it is clear that it is much more skewed towards zero than then one of $\rho^{\text {down }}$, and with a posterior average of 0.25 against 0.57 of $\rho^{\text {down }}$

Concerning the fiscal coefficients, we find that announced tax rises, $\tau_{a}$, and future spending cuts, $\gamma_{f}$, exhibit a statistical significant recessionary effect, while the other shocks do not. Their posterior probability of being negative is $92 \%$ and $97 \%$ respectively. Interestingly, the effect of announced spending cuts, $\gamma_{a}$, is statistically significant and expansionary, or positive.
Nevertheless, the single coefficients of the three components of fiscal adjustment plans are not very informative: we are interested in the convex combi-

[^13]Table II: Estimation Results

| Baseline Model - Equation (6) |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Parameters | MLE |  | Bayesian MCMC - Posterior Distributions: |  |  |  |  |  |  |  |  |  |
|  | $\hat{\theta}_{i}^{\mathrm{ML}}$ | MLE Std. | $\mathbb{E}\left(\theta_{i}\right)$ | $\sqrt{\mathbb{V}\left(\theta_{i}\right)}$ | $\operatorname{Pr}\left(\theta_{i}<0\right)$ | 5\% | 10\% | 16\% | 50\% | 84\% | 90\% | 95\% |
| $\rho^{\text {down }}$ (TB) | 0.603 | 0.125 | 0.569 | 0.117 | 0.000 | 0.374 | 0.419 | 0.453 | 0.569 | 0.687 | 0.720 | 0.761 |
| $\tau_{u}$ | 0.411 | 1.278 | 0.555 | 1.196 | 0.322 | -1.411 | -0.971 | -0.629 | 0.551 | 1.743 | 2.095 | 2.533 |
| $\tau_{a}$ | -1.259 | 0.990 | -1.294 | 0.930 | 0.917 | -2.820 | -2.488 | -2.218 | -1.295 | -0.366 | -0.100 | 0.237 |
| $\tau_{f}$ | -0.192 | 0.432 | -0.219 | 0.404 | 0.708 | -0.887 | -0.735 | -0.621 | -0.220 | 0.182 | 0.300 | 0.447 |
| $\rho^{u p}$ (EB) | 0.271 | 0.092 | 0.247 | 0.096 | 0.000 | 0.088 | 0.121 | 0.148 | 0.246 | 0.343 | 0.372 | 0.407 |
| $\gamma_{u}$ | -0.167 | 1.129 | -0.132 | 1.046 | 0.551 | -1.855 | -1.460 | -1.166 | -0.130 | 0.907 | 1.207 | 1.582 |
| $\gamma_{a}$ | 0.942 | 0.616 | 1.037 | 0.582 | 0.038 | 0.077 | 0.292 | 0.461 | 1.039 | 1.610 | 1.779 | 1.997 |
| $\gamma_{f}$ | -0.477 | 0.283 | -0.482 | 0.261 | 0.968 | -0.908 | -0.817 | -0.742 | -0.481 | -0.224 | -0.148 | -0.053 |
| D2008 | -2.941 | 0.671 | -2.903 | 0.633 | 1.000 | -3.946 | -3.714 | -3.532 | -2.902 | -2.274 | -2.092 | -1.861 |
| D2009 | -5.664 | 0.671 | -5.326 | 0.658 | 1.000 | -6.416 | -6.173 | -5.981 | -5.321 | -4.672 | -4.488 | -4.248 |

Table II: $\theta_{i}$ denotes a generic parameter that we estimate. The columns report the following: $\hat{\theta}_{i}^{M L}$ is the ML point estimate; "MLE Std." is the standard deviation of the ML estimate, calculated using the analytical Fisher Information Matrix derived in Appendix C.2: $\sqrt{\mathscr{I}\left(\hat{\theta}^{M L}\right)_{i i}^{-1}} ; \mathbb{E}\left(\theta_{i}\right)$ is the expected value of the posterior distribution; $\sqrt{\mathbb{V}\left(\theta_{i}\right)}$ is the standard deviation of the posterior distribution; $\operatorname{Pr}(\theta<0)$ is the probability that a parameter is negative, calculated by integrating the posterior distribution; $p \%$ is the p-th percentile of the posterior distribution. For brevity we don't report here the Industry Fixed Effects and the Industry specific variances. We also include year dummies for 2008 and 2009 to improve the precision of our estimates by capturing the industry-wide dip caused by the Great Recession. In the first columns, the spatial parameters also report the type of fiscal plan they are interacted with (in blue).
nation of all three components in a fiscal plan. Similarly, the mere size of the spatial coefficients is not enough to quantify the aggregate direct and network effect. We address these issues in the following section.

### 3.2 Aggregate Output Effect of Fiscal Consolidations

We are interested in estimating the average aggregate output effect of fiscal consolidations and then breaking it down into its direct and network effect. Our spatial econometric methodology conveniently provides such a decomposition.
Firstly, fiscal consolidations are made of three components: unanticipated, anticipated, and future. Therefore, we cannot define the impulse response in the standard way as the the partial derivative of a dependent variable with respect to a single shock. Rather, we construct the impulse response as a convex combination of the individual derivatives of $\Delta \log \boldsymbol{y}_{t}$ with respect to each of the three components of fiscal consolidations. The weights on each component are determined by the "style" of the plan, defined analytically as:

$$
\underbrace{\boldsymbol{s}_{\boldsymbol{T B}}}_{3 \times 1}:=\left[\begin{array}{lll}
s_{T B}^{u} & s_{T B}^{a} & s_{T B}^{f}
\end{array}\right]^{T} \quad \underbrace{\boldsymbol{s}_{\boldsymbol{E B}}}_{3 \times 1}:=\left[\begin{array}{lll}
s_{E B}^{u} & s_{E B}^{a} & s_{E B}^{f}
\end{array}\right]^{T} .
$$

For instance, if we want to simulate the effects of a TB fiscal plan which is $30 \%$ unanticipated, $0 \%$ anticipated, and $70 \%$ future, then we would set: $s_{T B}^{u}=.3$,
$s_{T B}^{a}=0, s_{T B}^{f}=.7$ and the vector of the "style" would be: $\boldsymbol{s}_{T B}=\left[\begin{array}{ll}.3 & 0\end{array} .7\right]^{T}$.
Secondly, given: $i$. the above definition of impulse response, $i i$. the vector representation of Equation (6), iii. the vectors of fiscal parameters $\boldsymbol{\tau}^{T}=$ $\left[\begin{array}{lll}\tau_{u} & \tau_{a} & \tau_{f}\end{array}\right]$ and $\gamma^{T}=\left[\gamma_{u} \gamma_{a} \gamma_{f}\right]$ and iv. industry weights for TB plans $\boldsymbol{\Psi}^{T}=$ $\left[\psi_{1} \ldots \psi_{n}\right]$ and EB plans $\boldsymbol{\Lambda}^{T}=\left[\lambda_{1} \ldots \lambda_{n}\right]$; then, the $n \times 1$ vector of industry specific Total Effect of a TB plan $\left(T B_{t}=1\right.$ and $\left.E B_{t}=0\right)$ is defined as:

$$
\begin{aligned}
T E_{T B} & :=\left.s_{T B}^{u} \cdot \frac{\partial \Delta \log \boldsymbol{y}_{t}}{\partial f_{t}^{u}}\right|_{T B_{t}=1}+\left.s_{T B}^{a} \cdot \frac{\partial \Delta \log \boldsymbol{y}_{t}}{\partial f_{t}^{a}}\right|_{T B_{t}=1}+\left.s_{T B}^{f} \cdot \frac{\partial \Delta \log \boldsymbol{y}_{t}}{\partial f_{t}^{f}}\right|_{T B_{t}=1} \\
& =\underbrace{\left(I_{n}-\rho^{d o w n} \cdot A\right)^{-1}}_{:=\boldsymbol{H}^{T B}} \cdot \boldsymbol{\Psi} \cdot \boldsymbol{\tau}^{T} \cdot \boldsymbol{s}_{\boldsymbol{T B}}=\underbrace{\mathbf{H}^{T B} \cdot \boldsymbol{\Psi}}_{n \times 1} \cdot \underbrace{\boldsymbol{\tau}^{T} \cdot \boldsymbol{s}_{\boldsymbol{T B}}}_{1 \times 1}
\end{aligned}
$$

Analogously, for an EB plan we have:

$$
T E_{E B}:=\underbrace{\left(I_{n}-\rho^{u p} \cdot \hat{A}_{0}^{T}\right)^{-1}}_{:=\boldsymbol{H}^{E B}} \cdot \boldsymbol{\Lambda} \cdot \gamma^{T} \cdot \boldsymbol{s}_{\boldsymbol{E} \boldsymbol{B}}=\underbrace{\mathbf{H}^{E B} \cdot \boldsymbol{\Lambda}}_{n \times 1} \cdot \underbrace{\gamma^{T} \cdot \boldsymbol{s}_{\boldsymbol{E B}}}_{1 \times 1} .
$$

Using the spatial framework, we can break down the TE into a Direct and Network Effect, as in Acemoglu, Akcigit, and Kerr (2016) and Ozdagli and Weber (2017). The former represents the direct impact of the fiscal plan and the latter represents the network spillovers:

$$
\begin{array}{ll}
D E_{T B}=\boldsymbol{\Psi} \cdot \boldsymbol{\tau}^{T} \cdot \boldsymbol{s}_{\boldsymbol{T B}} & N E_{T B}=\left(\boldsymbol{H}^{T B}-I_{n}\right) \cdot \boldsymbol{\Psi} \cdot \boldsymbol{\tau}^{T} \cdot \boldsymbol{s}_{\boldsymbol{T B}} \\
D E_{E B}=\boldsymbol{\Lambda} \cdot \boldsymbol{\gamma}^{T} \cdot \boldsymbol{s}_{\boldsymbol{E B}} & N E_{E B}=\left(\boldsymbol{H}^{E B}-I_{n}\right) \cdot \boldsymbol{\Lambda} \cdot \boldsymbol{\gamma}^{T} \cdot \boldsymbol{s}_{\boldsymbol{E B}} .
\end{array}
$$

The TE, DE and NE are $n \times 1$ vectors of industry specific effects of fiscal adjustment plans. However, we are interested in their aggregate effect. Therefore, we take a weighted average across industries with weights given by each industry's output share. ${ }^{19}$ By doing so we obtain the Average Total Effect, ATE, of a fiscal consolidation. We similarly construct the Average Direct Effect, $A D E$, and the Average Network Effect, $A N E$. Notice that given the linearity of the weighted average operation, we have that $A T E=A D E+A N E$, which therefore summarizes the breakdown of the total effect into its two components.

Table III reports descriptive statistics of the posterior distributions of the $A T E$ and its decomposition into $A D E$ and $A N E$ for 2-year fiscal adjustment

[^14]plans in the United States. This is our main contribution to the literature on fiscal consolidations. We obtain these results via Monte-Carlo, by drawing the parameters of equation (6) from their estimated posterior distributions. ${ }^{20}$ The style of the simulated plans, $\boldsymbol{s}_{T B}$ and $\boldsymbol{s}_{E B}$ - which determines the composition of a fiscal plan in terms of unanticipated, anticipated, and future components - is randomly drawn at each iteration from a distribution which mimics the in-sample data and satisfies three conditions: 1) the overall size of a plan is $1 \% ; 2)$ the anticipated component is zero; 3) the horizon of the plan is two years. ${ }^{21}$ This procedure ensures that our results are robust to different styles of fiscal plans and are not driven by a style redistribution of the $1 \%$ fiscal shock.

Table III: Average Total, Direct and Network Effects of Fiscal Consolidations in the United States

| Baseline Model - Equation (6) |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbb{E}(\theta)$ | \% | $\sqrt{\mathbb{V}(\theta)}$ | $\operatorname{Pr}(\theta<0)$ | 5\% | 10\% | 16\% | 50\% | 84\% | 90\% | 95\% |
| $A T E_{T B}$ | -1.397 | 100\% | 1.109 | 0.904 | -3.297 | -2.835 | -2.487 | -1.346 | -0.308 | -0.027 | 0.328 |
| $A D E_{T B}$ | -1.017 | 73\% | 0.789 | 0.904 | -2.327 | -2.031 | -1.798 | -1.006 | -0.238 | -0.021 | 0.258 |
| $A N E_{T B}$ | -0.380 | 27\% | 0.337 | 0.904 | -1.014 | -0.825 | -0.694 | -0.328 | -0.066 | -0.006 | 0.065 |
| $A T E_{E B}$ | 0.370 | 100\% | 0.371 | 0.152 | -0.265 | -0.103 | 0.014 | 0.386 | 0.727 | 0.825 | 0.950 |
| $A D E_{E B}$ | 0.326 | 88\% | 0.327 | 0.152 | -0.225 | -0.088 | 0.012 | 0.336 | 0.643 | 0.732 | 0.845 |
| $A N E E_{E B}$ | 0.043 | $12 \%$ | 0.052 | 0.152 | -0.038 | -0.014 | 0.001 | 0.041 | 0.090 | 0.106 | 0.130 |

Table III: descriptive statistics of posterior distributions of Average Effects of a 2 years, $1 \%$ magnitude fiscal adjustment plan. 2 years means that results are calculated by cumulating the effect of the first year of the plan and then the second one. The style of the plan is simulated from a distribution which mimics the observed one; see Appendix C.3 for technical details. Columns: $\mathbb{E}(\theta)$ is the expected value of the posterior distribution; \% is the share of $A T E$ represented by $A D E$ and $A N E . \sqrt{\mathbb{V}(\theta)}$ is the standard deviations of the posterior distribution; $\operatorname{Pr}(\theta<0)$ is the probability of negative values, calculated by integrating the posterior distribution; "p\%" is the $p$-th percentile of the posterior distribution.

In Table III, we document two main facts. First of all, consistent with existing work, TB fiscal consolidations imply larger output losses than EB fiscal consolidations. The expected value of $A T E_{T B}$ is -1.397 against a positive and insignificant $A T E_{E B}$ of 0.370 . This implies that a 2 years TB fiscal consolidation of $1 \%$ causes a cumulative average contraction of $-1.397 \%$ over two years. On the other hand, the effects of EB fiscal consolidations are mildly positive and not statistically significant.
Secondly, around $27 \%$ of $A T E_{T B}$ comes from network spillovers, confirming the relevance of the industrial network in the transmission of the TB fiscal adjustments. On the contrary, the network propagation of an EB fiscal plan is much

[^15]smaller, accounting for only $12 \%$ of $A T E_{E B}$. We calculate the average extent to which differences in the network effects of EB and TB plans account for differences in their total effects: $\left|\mathbb{E}\left(A N E_{T B}\right)-\mathbb{E}\left(A N E_{E B}\right)\right| /\left|\mathbb{E}\left(A T E_{T B}\right)-\mathbb{E}\left(A T E_{E B}\right)\right|$. We find a value of approximately $25 \% .^{22}$ Therefore, we conclude that at least $25 \%$ of the difference between EB and TB output effects can be explained by differences in production network spillovers.

We summarize our findings so far. TB fiscal consolidations have stronger effects in the United States than EB fiscal consolidations, with an average two years contraction of around $-1.4 \%$. EB fiscal consolidations in the United States have effects which are either not statistically different from zero, or mildly expansionary after two years. ${ }^{23}$ Network effects of TB consolidations explain $27 \%$ of the overall contraction. On average, $25 \%$ of the differences in the ATE of TB and EB plans can be attributed to the stronger network propagation of TB fiscal consolidations.

## 4 Robustness

### 4.1 Spatial Model and Orders of Propagation

An alternative to spatial lags in our econometric model is a standard panel data model with several "cross-terms" representing the first-order, secondorder, and higher-order degrees of connection, as in Hale, Kapan, and Minoiu (2019). However, this methodology requires a large number of parameters to be estimated, especially when the network is persistent, and when higher-order propagation effects are relevant. On the contrary, a spatial variable is capable of capturing the entire feedback effect with an infinite number of orders of connection whose impact decays geometrically.
In order to assess whether the US industrial network with $n=62$ sectors generates relevant high-order spillovers, we perform the partitioning of the effect, similar to what suggested by LeSage and Pace (2009). For instance, for the downstream propagation, we have:

$$
\underbrace{\left(I_{n}-A\right)^{-1} \cdot \mathbf{1}_{n}}_{\text {Total Effect }}=\underbrace{\mathbf{1}_{n}}_{\text {Direct }}+\underbrace{A \cdot \mathbf{1}_{n}}_{\text {1st order In-degree }}+\underbrace{A^{2} \cdot \mathbf{1}_{n}}_{\text {2nd order In-degree }}+\ldots
$$

[^16]where the term in-degree refers to the fact that the row-sum of the elements of $A$ represents the weighted in-degree of the network (total share of input purchased by a sector). For the upstream propagation, we have:
$$
\underbrace{\left(I_{n}-\hat{A}^{T}\right)^{-1} \cdot \mathbf{1}_{n}}_{\text {Total Effect }}=\underbrace{\mathbf{1}_{n}}_{\text {Direct }}+\underbrace{\hat{A}^{T} \cdot \mathbf{1}_{n}}_{\text {1st order Out-degree }}+\underbrace{\left(\hat{A}^{T}\right)^{2} \cdot \mathbf{1}_{n}}_{\text {2nd order Out-degree }}+\ldots
$$
where the term out-degree refers to the fact that the row-sum of $\hat{A}^{T}$ represents the weighted out-degree of the network (total share of output sold to other sectors). ${ }^{24}$ By averaging across the 62 industries the above expressions, we can calculate how much of the average total effect (left hand side of the expressions) can be attributed to each order of propagation (addends of the right hand side of the expressions). The results are reported in Table IV

Table IV: Partitioning of the network

| Order | Downstream Network |  | Upstream Network |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\%$ | Cumulative | $\%$ | Cumulative |
| 0 (Direct) | $53.36 \%$ | $53.36 \%$ | $54.53 \%$ | $54.53 \%$ |
| $1^{\text {st }}$ | $24.53 \%$ | $77.89 \%$ | $23.34 \%$ | $77.86 \%$ |
| $2^{\text {nd }}$ | $11.49 \%$ | $89.39 \%$ | $11.33 \%$ | $89.20 \%$ |
| $3^{\text {rd }}$ | $5.48 \%$ | $94.87 \%$ | $5.52 \%$ | $94.72 \%$ |
| $4^{\text {th }}$ | $2.64 \%$ | $97.51 \%$ | $2.70 \%$ | $97.42 \%$ |
| $5^{\text {th }}$ | $1.28 \%$ | $98.79 \%$ | $1.32 \%$ | $98.74 \%$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |

Notice that, consistent with Acemoglu, Carvalho, et al. (2012) and Carvalho (2007), the first two orders of the in-degrees and out-degrees are enough to capture most of the spillovers, roughly $89 \%$ of the overall effects. However, to capture the whole scope of network effects we should add terms up to the 5th order, which account for almost $99 \%$ of the total effect. Since we have 6 "core regressors" (TB and EB unanticipated, announced, and future components), the adoption of cross terms which capture the order of propagation, would require us to include 6 times 5 orders plus one (the Direct effect) for a total of 36 core regressors. Considering this unfeasible econometric specification, we opt for the more parsimonious spatial lag.

[^17]
### 4.2 Dynamics and Delayed Network Effects

The baseline model specified by Equation (6) does not include any time lag. We adopt a fully static specification because annual industry value-added growth rates are not very persistent, in particular at the fine disaggregation level of 62 sectors. Nevertheless, few sectors still show a non-negligible degree of autocorrelation. Therefore, we check whether our results are robust to the inclusion of a lagged dependent variable and we augment Equation (6) with a time lag: $\phi_{i} \cdot \Delta \log y_{i, t-1}$ The results are summarized by cumulative dynamic ATE, ADE and ANE, which now take the form of cumulative impulse response functions, reported in Figure 8. The values of the median of the dynamic ATE, ADE and ANE (blue solid lines in Figure 8) are reported in Table V.
Notice that after year 2, the end of the fiscal consolidation, the dynamic
Figure 8: Cumulative Impulse Response Functions


Figure 8: blue solid lines are the median cumulative impulse response functions (median of the posterior distributions). Red dashed lines are the $5^{t h}$ and $95^{t h}$ percentile of their posterior distributions, which represent our confidence bands. The "shock" is constructed by simulating a two years fiscal adjustment plan of $1 \%$ of GDP, exactly as done earlier to derive our static baseline results.
response is minimal, which corroborates our static analysis. In general, the effects are slightly larger in year 2 compared to the ones estimated in the static model and reported in Table III. Except for this, the results are comparable: 1)

Table V: Median Cumulative Impulse Response Functions

|  | 1 year | \% | 2 years | \% | $\cdots$ | Long Run | \% |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A T E_{T B}$ | -0.695 | $100 \%$ | -1.865 | $100 \%$ | $\cdots$ | -2.351 | $100 \%$ |
| $A D E_{T B}$ | -0.526 | $76.7 \%$ | -1.403 | $75.2 \%$ | $\cdots$ | -1.683 | $71.6 \%$ |
| $A N E_{T B}$ | -0.162 | $23.3 \%$ | -0.445 | $24.8 \%$ | $\cdots$ | -0.644 | $28.4 \%$ |
| $A T E_{E B}$ | -0.523 | $100 \%$ | 0.486 | $100 \%$ | $\cdots$ | 0.628 | $100 \%$ |
| $A D E_{E B}$ | -0.472 | $90.2 \%$ | 0.433 | $89.1 \%$ | $\cdots$ | 0.573 | $91.2 \%$ |
| $A N E_{E B}$ | -0.049 | $9.8 \%$ | 0.041 | $10.9 \%$ | $\cdots$ | 0.063 | $8.8 \%$ |

TB fiscal consolidations are recessionary and statistically different from zero; 2) the network effect is around one-fourth of the total effect of a TB plan; 3) EB fiscal consolidations have a minor network effect in the order of $10 \%$ of the total effect; 4) EB fiscal consolidations seem to be expansionary, but nothing can be concluded since they are not statistically different from zero.

We conclude this section by highlighting one fact: from Table V we notice that the relevance of $A N E_{T B}$ increases over time, from $23.3 \%$ to $28.4 \%$ in the long-run. This could be indicative of delayed network effects. Suppose a price shock takes longer than a year to travel from one sector to another, then the relevance of the network effect will increase over time since the spillover takes time to kick-in. For instance, Smets, Tielens, and Van Hove (2019) show that the autocorrelation between inflation in crude oil's price and synthetic rubber's price spikes after three months. Then the autocorrelation between inflation in synthetic rubber's price and tires' price also spikes after three months, but the autocorrelation between inflation in tires' price and transport costs spikes after 16 months. Therefore, downstream propagation of price changes does seem to have delayed effects consistent with the increasing relevance over time of the network effect of TB fiscal adjustments. We leave the issue of timing of the network effect for future research.

### 4.3 Inverted Propagation Mechanism

The baseline regression equation, Equation (6), implicitly assumes that TB fiscal consolidations exclusively propagate downstream, from suppliers to customers while the opposite is true for EB fiscal consolidations. This is done by interacting $T B_{t}$ with $\Delta y_{i, t}^{\mathrm{down}}$ and $E B_{t}$ with $\Delta y_{i, t}^{\mathrm{up}}$. This assumption is consistent with the theoretical propagation suggested by the model.

We now relax the assumption above and we switch the interaction of our dummies with the spatial variables. Therefore, we estimate the following equation:

$$
\begin{align*}
\Delta \log y_{i, t} & =\tilde{\alpha}_{i}+(\tilde{\rho}^{\text {down }} \cdot \Delta y_{i, t}^{u p}+\psi_{i} \cdot(\underbrace{\tilde{\tau}_{u} \cdot f_{t}^{u}+\tilde{\tau}_{a} \cdot f_{t}^{a}+\tilde{\tau}^{f} \cdot f_{t}^{f}}_{\text {Tax Increases }})) \cdot T B_{t}+ \\
& +(\tilde{\rho}^{u p} \cdot \Delta y_{i, t}^{\text {down }}+\gamma_{i} \cdot(\underbrace{\tilde{\gamma}_{u} \cdot f_{t}^{u}+\tilde{\gamma}_{a} \cdot f_{t}^{a}+\tilde{\gamma}^{f} \cdot f_{t}^{f}}_{\text {Spending Cuts }})) \cdot E B_{t}+\tilde{\nu}_{i, t} \tag{7}
\end{align*}
$$

Another option is to consider all propagation channels at once by estimating a single larger model which nests both Equation (6) (baseline model) and (7) (inverted model). However, this option is intractable due to the large number of parameters relative to the sample size, and due to collinearity between the spatial variables. We therefore estimate two separate models and then we apply a Vuong test for non-nested models to see which one fits the data better (see Vuong (1989) and Wooldridge (2010)). We find that the theoretically consistent model of Equation (6), where TB shocks propagate downstream and EB shocks upstream, provides a better fit to the data but not enough to reject the null hypothesis of the Vuong test, which assumes that the two model describe the data equally well. ${ }^{25}$

Secondly, we use the new estimates from the inverted model of Equation (7) to calculate the total, direct and network effect. ${ }^{26}$ We find the network effect of EB plans accounts for only $6 \%$ of their total effect, against the $12 \%$ of the baseline model. On the contrary, the relevance of network effects of TB plans is basically unaffected, diminishing only by $1 \%$ relative to the baseline model (from $27 \%$ to $26 \%$ ). Moreover, its statistical significance declines, since the posterior distribution shrinks towards zero.

Overall, the results indicate that the baseline model, which is consistent with the theoretical transmission channel illustrated in Section 2.3, delivers slightly stronger network effects and a slightly better fit.

### 4.4 Spurious Correlation and Placebo Experiments

One result of the paper is to record significant network effects of TB fiscal consolidations, accounting for $27 \%$ of the total effect, and capable of explaining

[^18]up to one fourth of the differences between the total output effect of TB and EB fiscal consolidations. What feature of the network is at basis of such strong spillovers? Are we measuring spurious correlation between sectors? or are we capturing some deep structural feature of the industrial network?
First of all, we plot in Figure 9 the downstream network $A$ associated with the downstream propagation of TB fiscal consolidations. Recall that the generic element of $A$, denoted by $a_{i j}$, is given by the reliance of sector $i$ (row) on industrial input $j$ (column): SALES $S_{j \rightarrow i} / S A L E S_{i}$.

Figure 9: Small, medium and large elements of Downstream Network $A$


Figure 9 is a "threshold heat-map" which reports a blue cell if $a_{i j}<0.0001$, an orange cell if $a_{i j}>0.03$ and a white cell otherwise. ${ }^{27}$ Two facts are salient from this "X-ray" of the downstream network. Firstly, the columns of $A$ tend to contain either only very small or only very large values. Secondly, the

[^19]rows of $A$ do not exhibit such a pattern. In other words, some sectors, such as "Social Assistance" or "Motion Picture and Sound Recording Industries", produce an output that is either not employed at all as an intermediate by other sectors, or it is employed only in minor quantity. Unlike them, some other sectors, such as "Wholesale Trade" and "Miscellaneous Professional, Scientific and Technical Services", produce an output which is a key input of production for many sectors. The bottom line is that the US downstream network is characterized by the presence of key suppliers and the lack of key customers. This asymmetric nature of the I-O connections is a well-known feature in the production network literature (see Acemoglu, Carvalho, et al. (2012)).
An interesting robustness exercise is to see what happens to our estimates if we employ simulated network matrices that break this pattern. We estimate Equation (6) (baseline) several times by employing simulated downstream matrices ("placebo") and compare the results with the original estimates. We carry out two experiments:
i. Column-Shuffling: we randomly shuffle the order of the columns of $A$ and create 100 simulated downstream matrices. This random permutation of the columns allows us to break that natural equilibrium in which some sectors behave as key suppliers and others are marginalized. In fact, in this first simulation, some real-world key supplier might be forced to behave as a peripheral sector and vice-versa. Therefore we expect less statistically significant results.
ii. Row-Shuffling: we randomly shuffle the order of the rows of $A$. Unlike the first experiment, reshuffling the elements within a column (shuffle the order of the rows) does not break the aforementioned characterizing pattern of the US downstream network. Sectors that originally were key suppliers will still behave in the same way. The same is true for peripheral sectors. We are reshuffling elements with similar magnitude along a column of $A$. Therefore, we expect to record both stronger and weaker results in terms of statistical significance.

Notice that in a Bayesian framework it is not fully correct to talk about statistical significance, however, with a little abuse of terminology we state that the $A N E_{T B}$ is more statistically significant if the values of $\mathbb{E}\left(A T E_{T B}\right) / \sqrt{\mathbb{V}\left(A T E_{T B}\right)}$ and $\operatorname{Pr}\left(A T E_{T B}>0\right)$ are both smaller. The first measure represents how many standard deviations we need, to obtain the average $A N E_{T B}$ : the smaller it is, the more likely is to obtain sizable negative spillovers. The second measure is simply the probability of obtaining a non-negative network effect: the smaller it is, the higher the chances of getting recessionary spillovers.

Figure 10 plots on the horizontal axis the first measure and on the vertical axis the second one. The red dot represents the values obtained by employing the original matrix $A$ (see Table III). The left panel of Figure 10 reports the

Figure 10: Placebo Experiment on $A N E_{T B}$

results of the experiment of shuffling the order of the columns: the red-dot is located in the South-West region of the graph, indicative of more significant spillover effects, as expected. The right panel reports the results of the "rowshuffling" experiment: the red-dot is located almost in the middle of the cloud of simulations' results, also in line with what expected. ${ }^{28}$
We highlight that these three steps procedure (simulation of network matrices, re-estimation, and comparison with the original values) is analogous to Ozdagli and Weber (2017). Unlike them, our "placebo" matrices are simulated in a simpler way by simply reshuffling the orders of the columns and rows.
Our procedure has the benefit of preserving the original elements of the network matrices, thus matching one to one both the distribution of the original elements $a_{i j}$, as well as its sparsity (number of zero entries). Unlike the original network $A$, the placebo matrices do not have large entries on the main diagonal in either simulations ("dense main diagonal").
Concerning the first order weighted in-degrees $\left(A \cdot \mathbf{1}_{n}\right)$ we have that the placebo matrices will exactly match it in the first simulation (shuffling the columns) while in the second one (shuffling the rows), the values are the same but they are assigned to different industries.
The second-order weighted in-degrees $\left(A^{2} \cdot \mathbf{1}_{n}\right)$ are not matched in either simulation, but the shape of their distribution is similar to the original one. Table

[^20]VI summarizes the results.

Table VI: Placebo Experiment Results

| Network Features: | Shuffling the Columns | Shuffling the Rows |
| :--- | :---: | :---: |
| Sparsity | same | same |
| Distribution of $a_{i j}$ | same | same |
| Dense Main Diagonal | no | no |
| $1^{\text {st }}$ Weighted In-degree | same values | same distribution |
| $2^{\text {nd }}$ Weighted In-degree | similar distribution | similar distribution |
| Key Suppliers | same | different |
| Peripheral Suppliers | same | different |
| Is original $A N E_{T B}$ stronger? | yes | no |

Ozdagli and Weber (2017) conclude that matching the first and second order out-degree is not sufficient to justify the strong upstream propagation of monetary policy shocks. In fact, they say, matching the properties of the network industry by industry is necessary to obtain a strong network effect. We achieve the same conclusion in the context of downstream propagation of TB fiscal consolidations, measured by $A N E_{T B}$, by means of an easier experiment, namely shuffling the order of rows and columns.
Finally, we answer the initial two questions: the significant downstream network effect of TB fiscal consolidation that we find, is not capturing a spurious relationship between the sectors, otherwise its effects should not be stronger than the placebo ones when we shuffle the columns. In fact, the downstream propagation hinges on the presence of key suppliers of input of production in the industrial network, as witnessed by the lack of superior results when employing the original downstream matrix and we break this pattern (row shuffling).

## 5 Conclusions

This paper investigates the effects of fiscal consolidations and their propagation in the industrial network in the US from 1978-2014. We find that TB fiscal consolidations are associated with slower consumption growth and are implemented with excise/production tax increases which are supposed to propagate downstream in the production network, via price increases. EB fiscal consolidations have no recessionary effects and are implemented mainly with
procurement spending cuts which propagate upstream in the production network via changes in input-demand. Using a panel of 62 industries, we find evidence of network effects of fiscal consolidations. In particular, we apply spatial econometric techniques to break down the total aggregate effect of fiscal consolidations into a direct component and a network component.
Firstly, we find stronger effects of tax-based fiscal adjustments. In particular, an adjustment of one percent of GDP leads to an average contraction over two years of about $-1.4 \%$ of value-added. Secondly, $27 \%$ of this effect can be attributed to spillovers from a supplying industry to a customer one. Thirdly, we find no evidence for a statistically significant recessionary impact of fiscal consolidations achieved by means of spending cuts. Rather, our evidence indicates mild expansionary effects. Fourthly, only $11 \%$ of EB effects originate from an upstream network transmission. Fifthly, we find that almost one-fourth of the different average total effects of TB and EB fiscal consolidations can be explained by stronger network spillovers of the former. Moreover, placebo experiments find that such a network effect of TB fiscal plans originates from the presence of key suppliers in the economy and does not depend on the particular shape of the distribution of first and second-order in-degrees of the network. When those key suppliers are forced to behave as peripheral suppliers the downstream propagation of TB plans vanishes or becomes significantly weaker.
In terms of policy implications, we provide further evidence that a fiscal consolidation based on spending cuts should be preferred to one based on tax hikes. The rationale is that smaller negative spillovers associated with spending cuts reduce the overall output cost. Also, the placebo experiments stress the importance of key suppliers of input in the industrial network. However, we do not comment on the possibility of designing optimal policies which take into account the special role of key suppliers in the propagation of shocks. We plan to address these issues in further research.

## References

[1] Daron Acemoglu, Ufuk Akcigit, and William Kerr. "Networks and the macroeconomy: An empirical exploration." In: NBER Macroeconomics Annual 30.1 (2016), pp. 273-335. DOI: 10.1086/685961.
[2] Daron Acemoglu, Vasco Carvalho, Asuman Ozdaglar, and Alireza TahbazSalehi. "The network origins of aggregate fluctuations." In: Econometrica 80.5 (2012), pp. 1977-2016. DOI: 10.3982/ECTA9623.
[3] Alberto Alesina, Omar Barbiero, Carlo Favero, Francesco Giavazzi, and Matteo Paradisi. "The effects of fiscal consolidations: Theory and evidence." In: (2017). DOI: 10.3386/w23385.
[4] Alberto Alesina, Carlo Favero, and Francesco Giavazzi. Austerity: When it Works and when it Doesn't. Princeton University Press, 2020.
[5] Alberto Alesina, Carlo Favero, and Francesco Giavazzi. "The output effect of fiscal consolidation plans." In: Journal of International Economics 96 (2015), S19-S42. DOI: 10.1016/j.jinteco.2014.11.003.
[6] Michele Aquaro, Natalia Bailey, and M Hashem Pesaran. "Estimation and inference for spatial models with heterogeneous coefficients: an application to US house prices." In: USC-INET Research Paper 19-07 (2019).
[7] Alan J Auerbach, Yuriy Gorodnichenko, and Daniel Murphy. Local Fiscal Multipliers and Fiscal Spillovers in the United States. Tech. rep. National Bureau of Economic Research, 2019. Doi: 10.3386/w25457.
[8] David Rezza Baqaee and Emmanuel Farhi. "JEEA-FBBVA Lecture 2018: The Microeconomic Foundations of Aggregate Production Functions." In: Journal of the European Economic Association 17.5 (2019), pp. 13371392.
[9] David Rezza Baqaee and Emmanuel Farhi. Macroeconomics with heterogeneous agents and input-output networks. Tech. rep. National Bureau of Economic Research, 2018.
[10] David Rezza Baqaee and Emmanuel Farhi. "The macroeconomic impact of microeconomic shocks: beyond Hulten's Theorem." In: Econometrica 87.4 (2019), pp. 1155-1203.
[11] Jean-Noël Barrot and Julien Sauvagnat. "Input specificity and the propagation of idiosyncratic shocks in production networks." In: The Quarterly Journal of Economics 131.3 (2016), pp. 1543-1592.
[12] Christoph E Boehm, Aaron Flaaen, and Nitya Pandalai-Nayar. "Input linkages and the transmission of shocks: Firm-level evidence from the 2011 Tōhoku earthquake." In: Review of Economics and Statistics 101.1 (2019), pp. 60-75.
[13] H Bouakez, O Rachedi, and E Santoro. "The Government Spending Multiplier in a Multi-Sector Model." In: American Economic Journal: Macroeconomics (2020).
[14] H Bouakez, O Rachedi, and E Santoro. The sectoral origins of the spending multiplier. Tech. rep. Working paper, 2020.
[15] Edoardo Briganti and Victor Sellemi. "Who Anticipates Government Spending? Evidence from Defense Procurement." In: Working Paper (2022).
[16] Pedro Brinca, Miguel H Ferreira, Francesco Franco, Hans A Holter, and Laurence Malafry. "Fiscal consolidation programs and income inequality." In: International Economic Review 62.1 (2021), pp. 405-460.
[17] Vasco Carvalho. "Aggregate fluctuations and the network structure of intersectoral trade." In: (2007).
[18] Vasco Carvalho. "From micro to macro via production networks." In: Journal of Economic Perspectives 28.4 (2014), pp. 23-48. DOI: 10.1257/ jep.28.4.23.
[19] Vasco Carvalho and Alireza Tahbaz-Salehi. "Production networks: A primer." In: Annual Review of Economics 11 (2019), pp. 635-663.
[20] Fabrice Collard, Michel Habib, and Jean-Charles Rochet. "Sovereign debt sustainability in advanced economies." In: Journal of the European economic association 13.3 (2015), pp. 381-420.
[21] Lydia Cox, Gernot Muller, Ernesto Pasten, Raphael Schoenle, and Michael Weber. Big g. Tech. rep. National Bureau of Economic Research, 2020.
[22] Julian Di Giovanni and Galina Hale. Stock market spillovers via the global production network: transmission of US monetary policy. Tech. rep. National Bureau of Economic Research, 2021.
[23] George W Evans, Seppo Honkapohja, and Kaushik Mitra. "EXPECTATIONS, STAGNATION, AND FISCAL POLICY: A NONLINEAR ANALYSIS." In: International Economic Review (2022).
[24] Xavier Gabaix. "The granular origins of aggregate fluctuations." In: Econometrica 79.3 (2011), pp. 733-772. DOI: 10.3982/ECTA8769.
[25] Jaime Guajardo, Daniel Leigh, and Andrea Pescatori. "Expansionary austerity? International evidence." In: Journal of the European Economic Association 12.4 (2014), pp. 949-968. DOI: 10.1111/jeea. 12083.
[26] Galina Hale, Tumer Kapan, and Camelia Minoiu. "Shock transmission through cross-border bank lending: Credit and real effects." In: (2019). DOI: 10.17016/FEDS. 2019.052.
[27] Karen J Horowitz, Mark A Planting, et al. Concepts and Methods of the us input-Output Accounts. Tech. rep. Bureau of Economic Analysis, 2006.
[28] Madina Karamysheva. "How do fiscal adjustments work? An empirical investigation." In: Journal of Economic Dynamics and Control 137 (2022), p. 104347.
[29] Harry H Kelejian and Ingmar R Prucha. "A generalized moments estimator for the autoregressive parameter in a spatial model." In: International economic review 40.2 (1999), pp. 509-533.
[30] LungFei Lee. "Asymptotic distributions of quasi-maximum likelihood estimators for spatial autoregressive models." In: Econometrica 72.6 (2004), pp. 1899-1925.
[31] James LeSage and Robert Pace. "Introduction to spatial econometrics." In: (2009). DOI: 10.1111/j.1538-4632.2010.00797.x.
[32] James LeSage and Olivier Parent. "Bayesian model averaging for spatial econometric models." In: Geographical Analysis 39.3 (2007), pp. 241267. DOI: $10.1111 / \mathrm{j} .1538-4632.2007 .00703 . x$.
[33] Karel Mertens and Morten Ravn. "Empirical Evidence on the Aggregate Effects of Anticipated and Unanticipated US Tax Policy Shocks." In: American Economic Journal: Economic Policy 4.2 (2012), pp. 145-81.
[34] Christopher J Nekarda and Valerie A Ramey. "Industry evidence on the effects of government spending." In: American Economic Journal: Macroeconomics 3.1 (2011), pp. 36-59.
[35] Keith Ord. "Estimation methods for models of spatial interaction." In: Journal of the American Statistical Association 70.349 (1975), pp. 120126. DOI: 10.1080/01621459.1975.10480272.
[36] Ali Ozdagli and Michael Weber. "Monetary policy through production networks: Evidence from the stock market." In: (2017). DOI: 10.3386/ w23424.
[37] Ugo Panizza, Federico Sturzenegger, and Jeromin Zettelmeyer. "The economics and law of sovereign debt and default." In: Journal of economic literature 47.3 (2009), pp. 651-98.
[38] Roberto Perotti. "In search of the transmission mechanism of fiscal policy." In: NBER Macroeconomics Annual, Chicago University Press 22.1 (2007), pp. 169-226.
[39] Andrea Pescatori, Mr Daniel Leigh, Jaime Guajardo, and Mr Pete Devries. "A new action-based dataset of fiscal consolidation." In: 11-128 (2011). DOI: 10.5089/9781462346561.001.A001.
[40] Valerie A Ramey. "Ten years after the financial crisis: What have we learned from the renaissance in fiscal research?" In: Journal of Economic Perspectives 33.2 (2019), pp. 89-114.
[41] Valerie A Ramey and Matthew D Shapiro. Costly capital reallocation and the effects of government spending. Tech. rep. National Bureau of Economic Research, 1999.
[42] Carmen M Reinhart and Kenneth S Rogoff. This time is different: Eight centuries of financial folly. princeton university press, 2009.
[43] Christina D Romer and David H Romer. "The macroeconomic effects of tax changes: estimates based on a new measure of fiscal shocks." In: American Economic Review 100.3 (2010), pp. 763-801. DOI: 10.1257/ aer.100.3.763.
[44] Frank Smets, Joris Tielens, and Jan Van Hove. "Pipeline pressures and sectoral inflation dynamics." In: (2019). DOI: 10.2139/ssrn. 3346371.
[45] Erling Steigum and $\emptyset y s t e i n ~ T h ø g e r s e n . ~ " B o r r o w ~ a n d ~ a d j u s t: ~ F i s c a l ~ p o l-~$ icy and sectoral adjustment in an open economy." In: International Economic Review 44.2 (2003), pp. 699-724.
[46] Quang H. Vuong. "Likelihood ratio tests for model selection and nonnested hypotheses." In: Econometrica (1989). DOI: 10.2307/1912557.
[47] Jeffrey M Wooldridge. Econometric analysis of cross section and panel data. MIT press, 2010.
[48] Jihai Yu, Robert DeJong, and LungFei Lee. "Quasi-maximum likelihood estimators for spatial dynamic panel data with fixed effects when both n and T are large." In: Journal of Econometrics 146.1 (2008), pp. 118-134.
[49] Sarah Zubairy. "On fiscal multipliers: Estimates from a medium scale DSGE model." In: International Economic Review 55.1 (2014), pp. 169195.

## Appendices

## A Details on Aggregate Level Analysis

Impulse response functions are computed following the algorithm. Step 1 solve dynamically forward the estimated equation putting all shocks to zero; step 2 - simulate the equation setting the fiscal adjustment plan to $1 \%$ of GDP; step 3 - compute impulse response as a difference between step 2 and step 1; step 4 - compute confidence intervals using the block bootstrap to take into account serial correlation.

In Section 2.2 we build impulse response functions using truncated moving average model. In particular we estimate the following specification:

$$
\begin{aligned}
& \Delta y_{t}=\alpha+B_{1}(L) \cdot f_{t}^{u} \cdot E B_{t}+B_{2}(L) f_{t}^{u} \cdot T B_{t}+\ldots \\
& \quad \ldots+C_{1}(L) \cdot f_{t}^{a} \cdot E B_{t}+C_{2}(L) \cdot f_{t}^{a} \cdot T B_{t}+\ldots \\
& \ldots+\sum_{j=1}^{H} D_{j} \cdot f_{t, t+j}^{a} \cdot E B_{t}+\sum_{j=1}^{H} E_{j} \cdot f_{t, t+j}^{a} \cdot T B_{t}+\epsilon_{t}
\end{aligned}
$$

with:

$$
\begin{aligned}
& f_{t, t+j}^{a}=\delta_{j}^{T B} \cdot f_{t}^{u} \cdot T B_{t}+\epsilon_{t+j}^{1}, \text { for } j=\overline{1, H} \\
& f_{t, t+j}^{a}=\delta_{j}^{E B} \cdot f_{t}^{u} \cdot E B_{t}+\epsilon_{t+j}^{2}, \text { for } j=\overline{1, H}
\end{aligned}
$$

where $B(L)$ and $C(L)$ are polynomials of the length six, $H$ - is the anticipation horizon and also equal to six. We follow Mertens and Ravn (2012) on this and six is the median implementation lag.

Figure 11 and 12 show the estimated impulse response functions of several other tax receipts shares of GDP to TB and EB fiscal adjustment plans. Those impulse responses are obtained using truncated moving average model.

## B Industry Data

In this section we describe the data we use in our analysis.
Firstly, the disaggregation level, $n=62$, is determined by starting from the finest decomposition available on the Bureau of Economic Analysis (BEA) at a yearly frequency, namely 71 sectors, and then aggregating those sectors whose

Figure 11: Tax Receipts Response to TB Plans

data are not available for older years. We exclude the Government sector and consider only Government Enterprises as the only public, but politically independent, sector. The Government sector needs to be excluded since its outcome variable is G , government spending, which mechanically falls when a fiscal adjustment occurs.

Figure 12: Tax Receipts Response to EB Plans


## Value Added

We use real industry value-added as the dependent variable, $\Delta y_{i t}$. Value-added equals gross output minus intermediate inputs. It consists of compensation of employees, taxes on production and imports less subsidies (formerly indirect business taxes and non-tax payments), and gross operating surplus (formerly
"other value added"). We prefer it over gross output to be consistent with Acemoglu, Akcigit, and Kerr (2016). ${ }^{29}$

## Industry Specific Shares:

Following Acemoglu, Akcigit, and Kerr (2016), we construct the vector of industry-specific weights by exploiting information from the input-output tables, namely: $\omega_{i}^{E B}=\frac{\text { Sales }_{i \rightarrow G}}{\text { Sales }_{i}}$; where "G" stands for Government. ${ }^{30} \mathrm{By}$ doing so, we take into account the fact that the government purchases goods and services in different quantities from each sector. ${ }^{31}$ Lastly, the vector of weights for the EB plan, denoted by $\boldsymbol{\omega}^{E B}$, is then normalized to one.
On the contrary, we assume that aggregate TB fiscal plans impact each sector in the same fashion, therefore, we set $\omega_{i}^{T B}=1 / n$ for all $i$ and the $n \times 1$ vector will be: $\boldsymbol{\omega}^{T B}=1 / n \cdot \mathbf{1}_{n}$.

## B. 1 Input-Output Network

The BEA provides I-O tables that report the amount of commodity used (Use Table) and made (Make Table) by each industry. Horowitz, Planting, et al. (2006) outline the procedure to construct an industry-by-industry direct requirement matrix, with elements given by $S A L E S_{j \rightarrow i} / S A L E S_{i}$ for each sector. Let's denote this matrix by $A$ and note that its elements coincide one to one with the weights of $\Delta y_{i, t}^{\text {down }}$ in Equation 6. Therefore, the downstream spatial variable can be written in vector notation as: $\Delta \boldsymbol{y}_{t}^{\text {down }}=A \cdot \Delta \log \boldsymbol{y}_{t}$ and matrix $A$ can be constructed from the Make and Use Tables of the BEA. ${ }^{32}$ Henceforth we will refer to matrix $A$ as the "downstream matrix".
Finally, we construct a new matrix starting from $A$ and using BEA's industry specific gross output, such that its $(i j)_{t h}$ element is represented by $S A L E S_{i \rightarrow j} / S A L E S_{i}$, which coincides one to one with the weights of $\Delta y_{i, t}^{\text {up }}$ in Equation 6. We denote this new matrix by $\hat{A}^{T}$, and refer to it as the "upstream matrix". The upstream spatial variable can now be written in vector

[^21]notation as: $\Delta \boldsymbol{y}_{t}^{\mathrm{up}}=\hat{A}^{T} \cdot \Delta \log \boldsymbol{y}_{t}$.
The construction of matrices $A$ and $\hat{A}^{T}$ starts from the analysis of the Make and Use tables illustrated in chapter 12 of Horowitz, Planting, et al. (2006). We outline here the details of the construction and the precise mapping between the theory and the data.

## The Use Table

The Use table is a commodity-by-industry table which illustrates the uses of commodities by intermediate and final users. The rows of the Use Table represent the commodities (or products) and the sum of the entries in a row is the total output of that commodity. On the contrary, the columns display the industries that employ them and the final users. Horowitz, Planting, et al. (2006) provides a useful numerical example with 3 industries:

|  | Example of Use Table - 3 Industries |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Commodity/Industry | 1 | 2 | 3 | Final demand | Total Commodity Output |
| 1 | 50 | 120 | 120 | 40 | 330 |
| 2 | 180 | 30 | 60 | 130 | 400 |
| 3 | 50 | 150 | 50 | 20 | 270 |
| Scrap | 1 | 3 | 1 | 0 | 5 |
| VA | 47 | 109 | 34 | $/$ | 190 |
| Total Industry Output | 328 | 412 | 265 | 190 | $/$ |

What is of our interest is clearly the $n \times n$ commodity-by-industry part of the Table, whose values can be denoted with the following notation:

$$
(U s e)_{i j}=\mathrm{INP}_{i \rightarrow j}:=\text { Commodity } i \text { used as input by Industry } j
$$

Therefore, the $n \times n$ part of the Use Table we are going to use is:

$$
U=\left[\begin{array}{lll}
\mathrm{INP}_{1 \rightarrow 1} & \mathrm{INP}_{1 \rightarrow 2} & \mathrm{INP}_{1 \rightarrow 3} \\
\mathrm{INP}_{2 \rightarrow 1} & \mathrm{INP}_{2 \rightarrow 2} & \mathrm{INP}_{2 \rightarrow 3} \\
& & \\
\mathrm{INP}_{3 \rightarrow 1} & \mathrm{INP}_{3 \rightarrow 2} & \mathrm{INP}_{3 \rightarrow 3}
\end{array}\right]=\left[\begin{array}{ccc}
50 & 120 & 120 \\
180 & 30 & 60 \\
50 & 150 & 50
\end{array}\right]
$$

In practice, the above matrix $U$ is a "symmetric" commodity-by-industry Use Table.

Next step boils down in constructing a commodity-by-industry direct requirement table by dividing each industry's input, INP $_{j \rightarrow i}$, by its corresponding total industry output, $y_{i}$. We denote such a matrix with letter B:

$$
\mathrm{B}=\left[\begin{array}{ccc}
\frac{\mathrm{INP}_{1 \rightarrow 1}}{y_{1}} & \frac{\mathrm{INP}_{1 \rightarrow 2}}{y_{2}} & \frac{\mathrm{INP}_{1 \rightarrow 3}}{y_{3}} \\
\frac{\mathrm{INP}_{2 \rightarrow 1}}{y_{1}} & \frac{\mathrm{INP}_{2 \rightarrow 2}}{y_{2}} & \frac{\mathrm{INP}_{2 \rightarrow 3}}{y_{3}} \\
\frac{\mathrm{INP}_{3 \rightarrow 1}}{y_{1}} & \frac{\mathrm{INP}_{3 \rightarrow 1}}{y_{2}} & \frac{\mathrm{INP}_{3 \rightarrow 3}}{y_{3}}
\end{array}\right]=\left[\begin{array}{ccc}
\frac{50}{328} & \frac{120}{412} & \frac{120}{265} \\
\frac{180}{328} & \frac{30}{412} & \frac{60}{265} \\
\frac{50}{328} & \frac{150}{412} & \frac{50}{265}
\end{array}\right]=\left[\begin{array}{ccc}
0.152 & 0.291 & 0.453 \\
0.549 & 0.073 & 0.226 \\
0.152 & 0.364 & 0.189
\end{array}\right] .
$$

Notice one important thing: matrix $B$ is different from matrix $A$, since $x_{i \rightarrow j} \neq$ $\mathrm{INP}_{i \rightarrow j}$ : the former is an industry output flow, while the second measures a commodity flow to an industry.

## The Make Table

The Make table is an industry-by-commodity table which shows the production of commodities by industries. Row $i$ represents an industry and its summation delivers the total industry output, $y_{i}$. Column $j$ represents a commodity and its summation delivers the total commodity output.
Borrowing again Horowitz, Planting, et al., 2006's 3 industries example, we have:

| Example of Make Table - 3 Industries |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Industry/Commodity | 1 | 2 | 3 | Scrap | Total Industry Output |
| 1 | 300 | 25 | 0 | 3 | 328 |
| 2 | 30 | 360 | 20 | 2 | 412 |
| 3 | 0 | 15 | 250 | 0 | 265 |
| Total Commodity Output | 330 | 400 | 270 | 5 | $/$ |

Similarly to what done for the Use Table, we are interested in the central $n \times n$ elements of the table, which we can denote by V . The generic element of the "heart" of the Make table is:

$$
(\text { Make })_{i j}=\mathrm{OUT}_{i \rightarrow j}:=\text { Commodity } j \text { produced by Industry } i
$$

Therefore, the $n \times n$ part of the Make Table we are going to employ is:

$$
V=\left[\begin{array}{ccc}
\mathrm{OUT}_{1 \rightarrow 1} & \mathrm{ouT}_{1 \rightarrow 2} & \mathrm{OUT}_{1 \rightarrow 3} \\
\mathrm{ouT}_{2 \rightarrow 1} & \mathrm{ouT}_{2 \rightarrow 2} & \mathrm{ouT}_{2 \rightarrow 3} \\
\mathrm{OUT}_{3 \rightarrow 1} & \mathrm{ouT}_{3 \rightarrow 2} & \mathrm{ouT}_{3 \rightarrow 3}
\end{array}\right]=\left[\begin{array}{ccc}
300 & 25 & 0 \\
30 & 360 & 20 \\
0 & 15 & 250
\end{array}\right]
$$

In practice, the above matrix $V$ is a "symmetric" industry-by-commodity Make Table.

Analogously to what done before, we now take ratios; in particular, we divide each element of V by the total production of commodity $j$. The resulting matrix is denoted by D , and its generic element is:

$$
(D)_{i j}=\frac{O U T_{i \rightarrow j}}{\sum_{k=1}^{n} O U T_{k \rightarrow j}}=\frac{O U T_{i \rightarrow j}}{C_{j}}
$$

where $C_{j}:=\sum_{k=1}^{n} O U T_{k \rightarrow j}$ is the total production of commodity $j$. D represents the share of industry $i$ in the total production of commodity $j$; not surprisingly, Horowitz, Planting, et al. (2006) refer to this matrix as the "market share matrix". In the 3 industries/commodities example we have:

$$
D=\left[\begin{array}{lll}
\frac{O U T_{1 \rightarrow 1}}{C_{1}} & \frac{O U T_{1 \rightarrow 2}}{C_{2}} & \frac{O U T_{1 \rightarrow 3}}{C_{3}} \\
\frac{O U T_{2 \rightarrow 1}}{C_{1}} & \frac{O U T_{2 \rightarrow 2}}{C_{2}} & \frac{O U T_{1 \rightarrow 3}}{C_{3}} \\
\frac{O U T_{3 \rightarrow 1}}{C_{1}} & \frac{O U T_{3 \rightarrow 2}}{C_{2}} & \frac{O U T_{3 \rightarrow 3}}{C_{3}}
\end{array}\right]=\left[\begin{array}{ccc}
\frac{300}{330} & \frac{25}{400} & \frac{0}{270} \\
\frac{30}{330} & \frac{360}{400} & \frac{20}{270} \\
\frac{0}{330} & \frac{15}{400} & \frac{250}{270}
\end{array}\right]=\left[\begin{array}{ccc}
0.909 & 0.063 & 0 \\
0.091 & 0.900 & 0.074 \\
0 & 0.038 & 0.926
\end{array}\right]
$$

## Adjustment for Scrap Products

The I-O accounts include a commodity for scrap, which is a byproduct of industry production. No industry produces scrap on demand; rather, it is the result of production to meet other demands. In order to make the I-O model work correctly, we have to eliminate scrap as a secondary product. At the same time, we must also keep industry output at the same level.

This adjustment is accomplished by calculating the ratio of non-scrap output to industry output for each industry and then applying these ratios to
the market shares matrix in order to account for total industry output. More precisely, the non-scrap ratio, which I denote by $\theta_{i}$, is defined as follows:

$$
\theta_{i}=\frac{y_{i}-(\mathrm{scrap})_{i}}{y_{i}}
$$

and represents the share of total industry output $i$ made of commodity different from "scrap". In the 3 industries example we have:

| Industry | Tot.Ind.Out. | Scrap | $\Delta$ | $\theta_{i}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 328 | 3 | 325 | 0.991 |
| 2 | 412 | 2 | 410 | 0.995 |
| 3 | 265 | 0 | 265 | 1 |

The market shares matrix, $D$, is adjusted for scrap by dividing each row by the non-scrap ratio for that industry. In the resulting transformation matrix, called W, the implicit commodity output of each industry has been increased. In other words, we are increasing each market share to take into account that to produce each unit of each commodity, industry $i$ will produce $1 / \theta_{i}$ units of output. In essence, we are spreading the production of commodity "scrap" over the production of all the other commodities:
$W=\left[\begin{array}{l}\frac{O U T_{1 \rightarrow 1}}{C_{1}} \cdot \frac{1}{\theta_{1}} \\ \frac{O U T_{1 \rightarrow 2}}{C_{2}} \cdot \frac{1}{\theta_{2}}\end{array} \frac{\frac{O U T_{1 \rightarrow 3}}{C_{3}} \cdot \frac{1}{\theta_{3}}}{\frac{O U T_{2 \rightarrow 1}}{C_{1}} \cdot \frac{1}{\theta_{1}}} \frac{\frac{O U T_{2 \rightarrow 2}}{C_{2}} \cdot \frac{1}{\theta_{2}}}{} \frac{\frac{O U T_{1 \rightarrow 3}}{C_{3}} \cdot \frac{1}{\theta_{3}}}{\frac{O U T_{3 \rightarrow 1}}{C_{1}} \cdot \frac{1}{\theta_{1}}} \frac{\frac{O U T_{3 \rightarrow 2}}{C_{2}} \cdot \frac{1}{\theta_{2}}}{} \frac{\frac{O U T_{3 \rightarrow 3}}{C_{3}} \cdot \frac{1}{\theta_{3}}}{}\left[\begin{array}{cccc}\frac{0.909}{0.991} & \frac{0.063}{0.991} & \frac{0}{0.991} \\ \frac{0.091}{0.995} & \frac{0.900}{0.995} & \frac{0.074}{0.995} \\ \frac{0}{1} & \frac{0.038}{1} & \frac{0.926}{1}\end{array}\right]=\left[\begin{array}{cccc}0.917 & 0.063 & \\ 0.091 & 0.904 & 0 \\ 0 & 0.038 & 0\end{array}\right.\right.$

## The Direct Requirement Table

To summarize:

1. We constructed matrix B , a commodity-by-industry direct requirement table, whose columns tell us how much an industry $j$ needs of commodity $i$ relative to its own total industry production.
2. We constructed matrix W , an industry-by-commodity matrix which represent the market share - adjusted for scrap - of each industry $i$ in the production of a commodity $j$.

By combining these two matrices we can obtain an industry-by-industry direct requirement matrix:


In order to understand the meaning of each element of matrix P , it is important to derive it analytically:

$$
P=\underbrace{\left[\begin{array}{lll}
\frac{O U T_{1 \rightarrow 1}}{C_{1} \cdot \theta_{1}} & \frac{O U T_{1 \rightarrow 2}}{C_{2} \cdot \theta_{2}} & \frac{O U T_{1 \rightarrow 3}}{C_{3} \cdot \theta_{3}} \\
\frac{O U T_{2 \rightarrow 1}}{C_{1} \cdot \theta_{1}} & \frac{O U T_{2 \rightarrow 2}}{C_{2} \cdot \theta_{2}} & \frac{O U T_{1 \rightarrow 3}}{C_{3} \cdot \theta_{3}} \\
\frac{O U T_{3 \rightarrow 1}}{C_{1} \cdot \theta_{1}} & \frac{O U T_{3 \rightarrow 2}}{C_{2} \cdot \theta_{2}} & \frac{O U T_{3 \rightarrow 3}}{C_{3} \cdot \theta_{3}}
\end{array}\right]}_{W} \underbrace{\left[\begin{array}{lll}
\frac{\mathrm{INP}_{1 \rightarrow 1}}{y_{1}} & \frac{\mathrm{INP}_{1 \rightarrow 2}}{y_{2}} & \frac{\mathrm{INP}_{1 \rightarrow 3}}{y_{3}} \\
\frac{\mathrm{INP}_{2 \rightarrow 1}}{y_{1}} & \frac{\mathrm{INP}_{2 \rightarrow 2}}{y_{2}} & \frac{\mathrm{INP}_{2 \rightarrow 3}}{y_{3}} \\
\frac{\mathrm{INP}_{3 \rightarrow 1}}{y_{1}} & \frac{\mathrm{INP}_{3 \rightarrow 2}}{y_{2}} & \frac{\mathrm{INP}_{3 \rightarrow 3}}{y_{3}}
\end{array}\right]}_{B}
$$

Denoting by $p_{i j}$ the generic element of P , we have:
$p_{i j}=\frac{\frac{O U T_{i \rightarrow 1}}{C_{1} \cdot \theta_{1}} \cdot \mathrm{INP}_{1 \rightarrow j}+\frac{O U T_{i \rightarrow 2}}{C_{2} \cdot \theta_{2}} \cdot \mathrm{INP}_{2 \rightarrow j}+\frac{O U T_{i \rightarrow 3}}{C_{3} \cdot \theta_{3}} \cdot \mathrm{INP}_{3 \rightarrow j}}{y_{j}} \approx \frac{\mathrm{SALES}_{i \rightarrow j}}{S A L E S_{j}}$
In other words, $p_{i j}$ represents how much industry $j$ depends on inputs form industry $i$ relative to its own total industry output $y_{j}$. ${ }^{33}$
Notice that the transposed of matrix P is approximately equal to matrix $A$ in the paper:

$$
P \approx\left[\begin{array}{ccc}
\frac{\mathrm{SALES}_{1 \rightarrow 1}}{\mathrm{SALES}_{1}} & \frac{\text { SALES }_{1 \rightarrow 2}}{\mathrm{SALES}_{2}} & \frac{\mathrm{SALES}_{1 \rightarrow 3}}{\mathrm{SALES}_{3}} \\
\frac{\mathrm{SALES}_{2 \rightarrow 1}}{\mathrm{SALES}_{1}} & \frac{\mathrm{SALES}_{2 \rightarrow 2}}{\mathrm{SALES}_{2}} & \frac{\mathrm{SALES}_{2 \rightarrow 3}}{\mathrm{SALES}_{3}} \\
\frac{\mathrm{SALES}_{3 \rightarrow 1}}{\mathrm{SALES}_{1}} & \frac{\mathrm{SALES}_{3 \rightarrow 2}}{\mathrm{SALES}_{2}} & \frac{\mathrm{SALES}_{3 \rightarrow 3}}{\mathrm{SALES}_{3}}
\end{array}\right] \Longrightarrow A \approx P^{T}
$$

[^22]Matrix $P$ can be either constructed from the Make and Use table or downloaded from the BEA, as an industry-by-industry direct requirement table. Its transposed value identifies the matrix $A$ in Equation (6).
The construction of matrix $\hat{A}^{T}$, in equation (7), is trivial once we have matrix A as well as a vector of average industry output.

## C Spatial Econometric Estimation

We believe that our empirical methodology presents some results of independent interest. Although we do not want to divert attention from the macroeconomic focus of the paper, we believe certain econometric facts are worth mentioning here in the Appendix. We provide this discussion in the spirit of promoting the usage of these new techniques in macroeconomic analysis.

Firstly, the adoption of spatial econometric methods allows us to disentangle the direct and network effect of aggregate shocks. This is a novel and recent innovation in macroeconomics, as noted in Ozdagli and Weber (2017). Secondly, spatial models are traditionally estimated by row-normalizing and removing the main diagonal from the weighting matrix. Another common assumption is homoskedasticity of the error term. In a recent paper, Aquaro, Bailey, and Pesaran (2019) develop a new estimator which relaxes homoskedasticity and allow for different spatial coefficients, thus indirectly relaxing the row-normalization assumption. They refer to it as Heterogenous Spatial Autoregressive model (HSAR). They also point out that not assuming zero entries on the main diagonal of the weighting matrix is simply a re-parameterization of the model, which does not harm the statistical properties of the MLE, but does change the interpretation of the parameters. ${ }^{34}$ Their econometric model, adopted by Ozdagli and Weber (2017), is very convenient for macroeconomic applications which use non-row-normalized, dense main diagonal weighting matrices and in a setting where units are subject to heteroskedastic idiosyncratic shocks.

However, we highlight that even the standard dynamic spatial panel autoregressive model of Yu, DeJong, and Lee (2008) can easily be relaxed to accommodate for non-zero entries on the main diagonal and non-row-normalized weighting matrix with heteroskedastic errors. ${ }^{35}$ Our construction of a Bayesian MCMC, similar to the one in LeSage and Pace (2009), is thus an easy and natural extension to the more general version of the spatial panel autoregressive

[^23]model of Yu, DeJong, and Lee (2008). Moreover, the Bayesian MCMC method provides an easy way to recover the posterior distributions of the aggregate effects of the shocks, as illustrated earlier.
We encourage macroeconomists to adopt spatial econometric tools to study the propagation of aggregate shocks into a network of sub-units (countries, industries, regions...) but in doing so we also recommend them to follow three good practices:

1. Firstly, always allow for heteroskedasticity, since sub-units in general have different volatilities.
2. Secondly, never remove the main diagonal from the empirically observed weighting matrices, in our case $A$ and $\hat{A}^{T}$. In fact, zero-entries in the main diagonal imposes a lack of spillovers within the same observed unit ("intra-unit feedback"). This is a reasonable assumption when units are individuals - like in standard spatial econometric applications - but it is not sensible when units are aggregates, such as industries. Notice, that the empirically observed $A$ and $\hat{A}^{T}$ weighting matrices from our analysis exhibit very dense main diagonals (see Figure 9).
3. Thirdly, never row-normalize the weighting matrices. Row-normalization flattens the differences in the degree of connection of each unit. For instance, in our application with the industrial network, $A$ and $\hat{A}^{T}$ exhibit very different row-sums, indicative of different degrees of exposure to customer and supplying industries.

We recommend using either the Bayesian MCMC methodology developed here and detailed in Appendix C. 3 or the HSAR model of Aquaro, Bailey, and Pesaran (2019), whenever the application requires heterogeneous spatial coefficients. The relationship between the two models is left for future research.

In what follows we outline the details of the spatial econometric estimator that we employ.

## C. 1 Log-likelihood

The standard way to estimate the parameters of Equations (6) and (7) is via maximum likelihood (see LeSage and Pace (2009) for an introduction to spatial econometrics). The asymptotic and small sample properties of the MLE have been studied in Lee (2004) for cross-sectional data, and in Yu, DeJong, and Lee (2008), for dynamic panel data models with fixed effects.

We provide here the derivation of the log-likelihood of the baseline model (6), necessary for the calculation of both the MLE and the conditional posterior distributions of the Bayesian MCMC. ${ }^{36}$ Collecting fiscal adjustment plans, industry fixed effects and other controls into matrix $X_{t}$, from Equation (6):

$$
\begin{aligned}
& H_{t}^{-1} \cdot \underset{n \times 1}{\Delta y_{t}}=\underset{n \times k}{X_{t}} \cdot \beta+\varepsilon_{t} \\
& H_{t}=\left(I_{n}-\rho^{\text {down }} \cdot A \cdot T B_{t}-\rho^{u p} \cdot \hat{A}^{T} \cdot E B_{t}\right)^{-1} \\
& \varepsilon_{t} \sim \mathscr{N}(0, \Omega), \forall t \in\{1, \ldots, T\} \\
& \Omega=\operatorname{diag}\left(\sigma_{1}^{2}, \ldots, \sigma_{n}^{2}\right) \\
& \varepsilon_{t} \perp \varepsilon_{t+i}, \quad \forall t \in\{1, \ldots, T\}, \forall i \in \mathscr{Z}
\end{aligned}
$$

where $k$ is the number of regressors. ${ }^{37}$ We now make a convenient change in the notation: 1. we now use ' as a symbol for transposition instead of ${ }^{T} ; 2$. we now set $\rho_{1}=\rho^{\text {down }}, \rho_{2}=\rho^{u p}, A=W_{1}$ and $\hat{A}^{\prime}=W_{2}$. We have:

$$
Z_{t}:=H_{t}^{-1} \cdot \Delta y_{t} \sim \mathscr{N}\left(X_{t} \beta, \Omega\right) \Longrightarrow \Delta y_{t} \sim \mathscr{N}\left(H_{t} X_{t} \beta, H_{t} \Omega H_{t}^{\prime}\right)
$$

The density function of the random vector $\Delta y_{t}$ is:
$f\left(\underset{n \times 1}{\Delta y_{t} \mid X_{t}}, \rho, \beta, \Omega\right)=\frac{1}{\sqrt{(2 \pi)^{n} \cdot\left|H_{t} \Omega H_{t}^{\prime}\right|}} \exp \left\{-\frac{1}{2} \cdot\left(\Delta y_{t}-H_{t} X_{t} \beta\right)^{\prime} \cdot\left(H_{t} \Omega H_{t}^{\prime}\right)^{-1} \cdot\left(\Delta y_{t}-H_{t} X_{t} \beta\right)\right\}$,
with $\rho=\left[\rho^{\text {down }}, \rho^{u p}\right]$.
Given that $\left(H_{t} \Omega H_{t}^{\prime}\right)^{-1}=\left(H_{t}^{\prime}\right)^{-1} \cdot \Omega^{-1} \cdot H_{t}^{-1}$ and $\left|H_{t} \Omega H_{t}^{\prime}\right|=\left|H_{t}\right|^{2} \cdot|\Omega|$, we have:

$$
\begin{aligned}
f\left(\Delta y_{t} \mid \cdot\right) & =(2 \pi)^{-n / 2} \cdot\left|H_{t}\right|^{-1} \cdot|\Omega|^{-1 / 2} \cdot \exp \left\{-\frac{1}{2}\left(Z_{t}-X_{t} \beta\right)^{\prime} \cdot H_{t}^{\prime} \cdot\left(H_{t}^{\prime}\right)^{-1} \cdot \Omega^{-1} \cdot H_{t}^{-1} \cdot H_{t} \cdot\left(Z_{t}-X_{t}\right)\right. \\
& =(2 \pi)^{-n / 2} \cdot\left|\left(I_{n}-\rho_{1} W 1 T B_{t}-\rho_{2} W_{2} E B_{t}\right)^{-1}\right|^{-1} \cdot|\Omega|^{-1 / 2} \exp \left\{-\frac{1}{2} \varepsilon_{t}^{\prime} \Omega^{-1} \varepsilon_{t}\right\} \\
& =(2 \pi)^{-n / 2} \cdot\left|I_{n}-\rho_{1} \cdot W_{1} \cdot T B_{t}-\rho_{2} \cdot W_{2} \cdot E B_{t}\right| \cdot|\Omega|^{-1 / 2} \exp \left\{-\frac{1}{2} \varepsilon_{t}^{\prime} \Omega^{-1} \varepsilon_{t}\right\},
\end{aligned}
$$

At this point we need to find the likelihood of the random vector $\Delta y=$ $\left[\begin{array}{lll}\Delta y_{1}^{\prime} & \ldots & \Delta y_{T}^{\prime}\end{array}\right]$. Since the model is static and we have assumed $\operatorname{cov}\left(\varepsilon_{t}, \varepsilon_{t-k}\right)=$

[^24]$\underset{n \times n}{\rho^{0}}$, then $\Delta y_{t}$ is iid over time. By consequence, the following holds:
\[

$$
\begin{aligned}
& f\left(\underset{n T \times 1}{\Delta y} \mid X_{1}, \ldots, X_{T}, \rho, \beta, \Omega\right)=\prod_{t=1}^{T} f\left(\underset{n \times 1}{\Delta y_{t} \mid} X_{t}, \rho, \beta, \Omega\right)=\left((2 \pi)^{n}|\Omega|\right)^{-T / 2} . \\
& \cdot \prod_{t=1}^{T}\left|I_{n}-\rho_{1} \cdot W_{1} \cdot T B_{t}-\rho_{2} \cdot W_{2} \cdot E B_{t}\right| \exp \left\{-\frac{1}{2} \cdot \sum_{t=1}^{T} \varepsilon_{t}^{\prime} \Omega^{-1} \varepsilon_{t}\right\} .
\end{aligned}
$$
\]

Now we divide the time series of length $T$ in three different sub-periods. In doing so, consider the following new parameters:

- $t_{1}$ : set of years when a tax based fiscal adjustment occurs.

Formally $t_{1}:=\left\{1, \ldots, t, \ldots, T_{1} \mid t\right.$ such that $\left.T B_{t}=1\right\}$. We set: $H_{t} \mid t \in$ $t_{1}=\left(I_{n}-\rho_{1} \cdot W_{1}\right)^{-1}=H_{\tau}$.

- $t_{2}$ : set of years when an expenditure tax based fiscal adjustment occurs. Formally: $t_{2}:=\left\{1, \ldots, t, \ldots, T_{2} \mid t\right.$ such that $\left.E B_{t}=1\right\}$. We set $H_{t} \mid t \in$ $t_{2}=\left(I_{n}-\rho_{2} \cdot W_{2}\right)^{-1}=H_{\gamma}$.
- $t_{3}$ : set of years when neither a tax based fiscal adjustment nor an expenditure based fiscal adjustment occurs.
Formally $t_{3}:=\left\{1, \ldots, t, \ldots, T_{3} \mid t\right.$ such that $\left.T B_{t}=0 \wedge E B_{t}=0\right\}$. We set $H_{t} \mid t \in t_{3}=\left(I_{n}\right)^{-1}=I_{n}$.

Therefore, we have that $t_{1}, t_{2}$ and $t_{3}$ account for a partition of the whole time series and $T=T_{1}+T_{2}+T_{3}$. By consequence we have:

$$
\begin{aligned}
\prod_{t=1}^{T}\left|I_{n}-\rho_{1} W_{1} T B_{t}-\rho_{2} W_{2} E B_{t}\right| & =\prod_{t=1}^{T}\left|H_{t}^{-1}\right| \\
& =\prod_{t=1}^{T} \frac{1}{\left|H_{t}\right|} \\
& =\prod_{t \in t_{1}}^{T_{1}} \frac{1}{\left|H_{t}\right|} \cdot \prod_{t \in t_{2}}^{T_{2}} \frac{1}{\left|H_{t}\right|} \cdot \prod_{t \in t_{3}}^{T_{3}} \frac{1}{\left|H_{t}\right|} \\
& =\left|H_{\tau}\right|^{-T_{1}} \cdot\left|H_{\gamma}\right|^{-T_{2}} \cdot\left|I_{n}\right|^{-T_{3}} \\
& =\left|I_{n}-\rho_{1} \cdot W_{1}\right|^{T_{1}} \cdot\left|I_{n}-\rho_{2} W_{2}\right|^{T_{2}}
\end{aligned}
$$

At this point, we rewrite the probability density function of our dependent
variable as:

$$
\begin{aligned}
& f\left(\Delta y_{t} \mid X_{1}, \ldots, X_{T}, \rho, \beta, \Omega\right)=(2 \pi)^{-n T / 2} \cdot|\Omega|^{-T / 2} \\
& \quad \cdot\left|I_{n}-\rho_{1} \cdot W_{1}\right|^{T_{1}} \cdot\left|I_{n}-\rho_{2} W_{2}\right|^{T_{2}} \cdot \exp \left\{-\frac{1}{2} \cdot \sum_{t=1}^{T} \varepsilon_{t}^{\prime} \cdot \Omega^{-1} \cdot \varepsilon_{t}\right\}
\end{aligned}
$$

Finally, the log-likelihood of our dataset is:

$$
\begin{aligned}
& \log \mathscr{L}\left(\rho, \beta, \Omega \mid \Delta y_{1}, \ldots, \Delta y_{T}, X_{1}, \ldots, X_{T}\right)=-\frac{n T}{2} \ln (2 \pi)-\frac{T}{2} \cdot \ln (|\Omega|)+ \\
& \quad+T_{1} \cdot \ln \left(\left|I_{n}-\rho_{1} \cdot W_{1}\right|\right)+T_{2} \cdot \ln \left(\left|I_{n}-\rho_{2} W_{2}\right|\right)-\frac{1}{2} \cdot \sum_{t=1}^{T} \varepsilon_{t}^{\prime} \cdot \Omega^{-1} \cdot \varepsilon_{t} .
\end{aligned}
$$

with:
$\varepsilon_{t}=Z_{t}-X_{t} \cdot \beta=H_{t}^{-1} \cdot \Delta y_{t}-X_{t} \beta=\left(I_{n}-\rho_{1} W_{1} T B_{t}-\rho_{2} W_{2} E B_{t}\right) \cdot \Delta y_{t}-X_{t} \cdot \beta$.
Furthermore, we impose the condition $\lambda_{\min }^{-1}<\hat{\rho}_{1}<\lambda_{\max }^{-1}$ and $\mu_{\min }^{-1}<\hat{\rho}_{2}<$ $\mu_{\max }^{-1}$, where $\lambda$ and $\mu$ are the eigenvalues of the spatial matrices $W_{1}$ and $W_{2}$ respectively. This condition guarantees that the estimated model will have positive definite covariance matrix (see Ord (1975)).
Notice that in the inverted model of Equation (7), it is enough to switch the definition of $W_{1}$ and $W_{2}$ by setting: $A=W_{2}$ and $\hat{A}^{\prime}=W_{1}$.

## C. 2 The Analytical Fisher Information Matrix

In order to derive the Fisher Information Matrix we firstly need to obtain the total gradient of the log-likelihood function. Let's start with the spatial coefficient $\rho_{1}$ :

$$
\frac{\partial \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \rho_{1}}=T_{1} \frac{1}{\left|I_{n}-\rho_{1} W_{1}\right|} \frac{\partial\left|I_{n}-\rho_{1} W_{1}\right|}{\partial \rho_{1}}-\frac{1}{2} \sum_{t=1}^{T} \frac{\partial\left(Z_{t}^{\prime} \Omega^{-1} Z_{t}\right)}{\partial \rho_{1}}-2 \frac{\partial\left(Z_{t}^{\prime} \Omega^{-1} X_{t} \beta\right)}{\partial \rho_{1}}
$$

By some matrix algebra, it is possible to show that:

$$
\begin{aligned}
\frac{\partial\left(Z_{t}^{\prime} \Omega^{-1} Z_{t}\right)}{\partial \rho_{1}} & =-T B_{t} \cdot \Delta y_{t}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}-T B_{t} \cdot \Delta y_{t}^{\prime} \cdot W_{1}^{\prime} \Omega^{-1} \cdot \Delta y_{t} \\
& +2 \rho_{1} \cdot T B_{t}^{2} \cdot \Delta y_{t}^{\prime} \cdot W_{1} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}^{\prime}+2 \rho_{2} \cdot T B_{t} \cdot E B_{t} \cdot \Delta y_{t}^{\prime} \cdot W_{1} \cdot \Omega^{-1} \cdot W_{2} \cdot \Delta y_{t}^{\prime}
\end{aligned}
$$

Since our fiscal adjustment plans are mutually exclusive, we have that $T B_{t}$. $E B_{t}=0$ for all $t$. Moreover, by rearranging the above expression, we get:

$$
\frac{\partial\left(Z_{t}^{\prime} \Omega^{-1} Z_{t}\right)}{\partial \rho_{1}}=-2 \cdot T B_{t} \cdot \Delta y_{t}^{\prime} \cdot\left(I_{n}-\rho_{1} \cdot W_{1}^{\prime}\right) \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}
$$

After other matrix algebra, we get:

$$
-2 \cdot \frac{\partial\left(Z_{t} \cdot \Omega^{-1} X_{t} \beta\right)}{\partial \rho_{1}}=2 \cdot T B_{t} \cdot \Delta y_{t}^{\prime} \cdot W_{1}^{\prime} \cdot \Omega^{-1} \cdot X_{t} \cdot \beta
$$

Wrapping up all together, and employing the notation introduced earlier: ( $I_{n}-$ $\left.\rho_{1} W_{1}\right)^{-1}=H_{\tau}$, we have:

$$
\begin{aligned}
\frac{\partial \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \rho_{1}} & =T_{1} \frac{1}{\left|I_{n}-\rho_{1} W_{1}\right|} \frac{\partial\left|I_{n}-\rho_{1} W_{1}\right|}{\partial \rho_{1}}+ \\
& +\sum_{t \in t_{1}}^{T_{1}}\left[\Delta y_{t}^{\prime} \cdot\left(I_{n}-\rho_{1} \cdot W_{1}^{\prime}\right) \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}-\Delta y_{t}^{\prime} \cdot W_{1}^{\prime} \cdot \Omega^{-1} \cdot X_{t} \cdot \beta\right]= \\
& =T_{1} \frac{1}{\left|I_{n}-\rho_{1} W_{1}\right|} \cdot\left|I_{n}-\rho_{1} W_{1}\right| \cdot \operatorname{Tr}\left(\left(I_{n}-\rho_{1} W_{1}\right)^{-1} \cdot\left(-W_{1}\right)\right)+ \\
& +\sum_{t \in t_{1}}^{T_{1}}\left[\left(\left(I_{n}-\rho_{1} \cdot W_{1}\right) \cdot \Delta y_{t}\right)^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}-\beta^{\prime} \cdot X_{t}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}\right] \\
& =-T_{1} \cdot \operatorname{Tr}\left(H_{\tau} \cdot W_{1}\right)+\sum_{t \in t_{1}}^{T_{1}}\left[\left(Z_{t}-X_{t} \beta\right)^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}\right] \\
& =\sum_{t \in t_{1}}^{T_{1}}\left(\varepsilon_{t}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}\right)-T_{1} \cdot \operatorname{Tr}\left(H_{\tau} \cdot W_{1}\right)
\end{aligned}
$$

By simmetry we have that:

$$
\frac{\partial \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \rho_{2}}=\sum_{t \in t_{2}}^{T_{2}}\left(\varepsilon_{t}^{\prime} \cdot \Omega^{-1} \cdot W_{2} \cdot \Delta y_{t}\right)-T_{2} \cdot \operatorname{Tr}\left(H_{\gamma} \cdot W_{2}\right)
$$

with $H_{\gamma}=\left(I_{n}-\rho_{2} W_{2}\right)^{-1}$, from the previous notation.
As far as concern the derivative with respect to $\beta$, we have already seen when
concentrating the log-likelihood that:

$$
\begin{aligned}
\frac{\partial \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \beta} & =X^{\prime} \cdot \Sigma^{-1} \cdot Z-X^{\prime} \cdot \Sigma^{-1} \cdot X \cdot \beta \\
& =X^{\prime} \cdot \Sigma^{-1} \cdot(Z-X \cdot \beta)= \\
& =X^{\prime} \cdot \Sigma^{-1} \cdot \varepsilon= \\
& =\sum_{t=1}^{T} X_{t}^{\prime} \cdot \Omega^{-1} \cdot \varepsilon_{t}
\end{aligned}
$$

Concerning the derivatives with respect to $\sigma_{i}^{2}$, we need firstly to acknowledge that:

$$
\sum_{t=1}^{T} \varepsilon_{t}^{\prime} \cdot \Omega^{-1} \cdot \varepsilon_{t}=\sum_{t=1}^{T} \sum_{i=1}^{n} \frac{\varepsilon_{i, t}^{2}}{\sigma_{i}^{2}}=\sum_{i=1}^{n} \frac{1}{\sigma_{i}^{2}} \sum_{t=1}^{T} \varepsilon_{i, t}^{2},
$$

and that:

$$
\ln (|\Omega|)=\ln \left(\prod_{i=1}^{n} \sigma_{i}^{2}\right)=\sum_{i=1}^{n} \ln \left(\sigma_{i}^{2}\right)
$$

Therefore, we have that:

$$
\begin{aligned}
\frac{\partial \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \sigma_{i}^{2}} & =-\frac{T}{2} \frac{\partial \ln (|\Omega|)}{\partial \sigma_{i}^{2}}-\frac{1}{2} \cdot \frac{\partial}{\partial \sigma_{i}^{2}} \sum_{t=1}^{T} \varepsilon_{t}^{\prime} \cdot \Omega^{-1} \cdot \varepsilon_{t} \\
& =-\frac{T}{2 \cdot \sigma_{i}^{2}}+\frac{1}{2 \cdot \sigma_{i}^{4}} \cdot \sum_{t=1}^{T} \varepsilon_{i, t}^{2}
\end{aligned}
$$

We now have all the elements to write down the gradient of the log-likelihood:
$\nabla \log \mathscr{L}(\theta \mid \Delta y, X)=\left[\begin{array}{c}\frac{\partial \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \rho_{1}} \\ \frac{\partial \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \rho_{2}} \\ \frac{\partial \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \beta} \\ \frac{\partial \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \sigma_{1}^{2}} \\ \vdots \\ \frac{\partial \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \sigma_{n}^{2}}\end{array}\right]=\left[\begin{array}{c}\sum_{t \in t_{1}}^{T_{1}}\left(\varepsilon_{t}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}\right)-T_{1} \cdot \operatorname{Tr}\left(H_{\tau} \cdot W_{1}\right) \\ \sum_{t \in t_{2}}^{T_{2}}\left(\varepsilon_{t}^{\prime} \cdot \Omega^{-1} \cdot W_{2} \cdot \Delta y_{t}\right)-T_{2} \cdot \operatorname{Tr}\left(H_{\gamma} \cdot W_{2}\right) \\ 38 \times 1\end{array}\right]$
Another round of derivation is now needed. Let's start with the first row of the matrix: all the derivatives of $\frac{\partial \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \rho_{1}}$ with respect to all the parameters. To simplify notation we will refer with $\mathscr{H}_{i j}$ to the element of row $i$ and column $j$ of the Hessian matrix.

$$
\begin{aligned}
\mathscr{H}_{1,1} & =\frac{\partial^{2} \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \rho_{1}^{2}}=\sum_{t \in t_{1}}^{T_{1}}\left(\frac{\partial \varepsilon_{t}^{\prime}}{\partial \rho_{1}} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}\right)-T_{1} \cdot \frac{\partial \operatorname{Tr}\left(H_{\tau} \cdot W_{1}\right)}{\partial \rho_{1}} \\
& =\sum_{t \in t_{1}}^{T_{1}}\left(\left(-\Delta y_{t}^{\prime} \cdot W_{1}^{\prime}\right) \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}\right)-T_{1} \cdot \operatorname{Tr}\left(\frac{\partial H_{\tau}}{\partial \rho_{1}} \cdot W_{1}\right)= \\
& =-\sum_{t \in t_{1}}^{T_{1}}\left(\Delta y_{t}^{\prime} \cdot W_{1}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}\right)-T_{1} \cdot \operatorname{Tr}\left(\left(-H_{\tau} \cdot\left(-W_{1}\right) \cdot H_{\tau}\right) \cdot W_{1}\right)= \\
& =-T_{1} \cdot \operatorname{Tr}\left(W_{1} \cdot H_{\tau} \cdot W_{1} \cdot H_{\tau}\right)-\sum_{t \in t_{1}}^{T_{1}}\left(\Delta y_{t}^{\prime} \cdot W_{1}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}\right)
\end{aligned}
$$

Symmetrically we have:

$$
\begin{aligned}
\mathscr{H}_{2,2} & =\frac{\partial^{2} \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \rho_{2}^{2}}= \\
& =-T_{2} \cdot \operatorname{Tr}\left(W_{2} \cdot H_{\gamma} \cdot W_{2} \cdot H_{\gamma}\right)-\sum_{t \in t_{2}}^{T_{2}}\left(\Delta y_{t}^{\prime} \cdot W_{2}^{\prime} \cdot \Omega^{-1} \cdot W_{2} \cdot \Delta y_{t}\right)
\end{aligned}
$$

Going back to the first row, we now calculate the cross derivative with respect to $\rho 2$. Before doing so, recall that, being the log-likelihood a continuously diffirentiable function, the Schwarz's theorem applies and the Hessian matrix is symmetric.

$$
\mathscr{H}_{1,2}=\mathscr{H}_{2,1}=\frac{\partial^{2} \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \rho_{1} \partial \rho_{2}}=0 .
$$

Going on with the calculation we have:

$$
\begin{aligned}
\mathscr{H}_{1,3: 1,23} & =\frac{\partial^{2} \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \rho_{1} \partial \beta}=\sum_{t \in t_{1}}^{T_{1}}\left(\frac{\partial \varepsilon_{t}^{\prime}}{\partial \beta} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}\right) \\
& =-\sum_{t \in t_{1}}^{T_{1}} X_{t}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t} \\
& =-X_{\tau}^{\prime} \cdot\left(\begin{array}{c}
\left.I_{T_{1}} \otimes \Omega^{-1}\right) \cdot\left(I_{T_{1}} \otimes W_{1}\right) \cdot \Delta y_{\tau}
\end{array}, \${ }_{\Sigma_{\tau}^{-1}}\right)
\end{aligned}
$$

where $\mathscr{H}_{1,3: 1,23}$ means all the elements of the first row, from column 3 up to column 23. $X_{\tau}$ and $\Delta y_{\tau}$ represent $X$ and $\Delta y$ but for the only years when a tax based fiscal adjustment occur:

$$
X_{\tau}=\underset{T_{1} n \times k}{\left[\begin{array}{c}
X_{1} \\
\vdots \\
X_{t} \\
\vdots \\
X_{T_{1}}
\end{array}\right] \quad \text { and } \quad \Delta y_{\tau}=\left[\begin{array}{c}
\Delta y_{1} \\
\vdots \\
\Delta y_{t} \\
\vdots \\
\Delta y_{T_{1}}
\end{array}\right] \quad \text { with } t \in t_{1}, .}
$$

Symmetrically:

$$
\begin{aligned}
\mathscr{H}_{2,3: 2,23} & =\frac{\partial^{2} \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \rho_{2} \partial \beta}=\sum_{t \in t_{2}}^{T_{2}}\left(\frac{\partial \varepsilon_{t}^{\prime}}{\partial \beta} \cdot \Omega^{-1} \cdot W_{2} \cdot \Delta y_{t}\right) \\
& =-\sum_{t \in t_{2}}^{T_{2}} X_{t}^{\prime} \cdot \Omega^{-1} \cdot W_{2} \cdot \Delta y_{t} \\
& =-X_{\gamma}^{\prime} \cdot\left(I_{T_{2}} \otimes \Omega^{-1}\right) \cdot\left(I_{T_{2}} \otimes W_{2}\right) \cdot \Delta y_{\gamma},
\end{aligned}
$$

with:

$$
\begin{aligned}
\mathscr{H}_{3,3: 23,23} & =\frac{\partial^{2} \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \beta^{2}}=\frac{\partial}{\partial \beta^{2}}\left(\sum_{t=1}^{T} X_{t}^{\prime} \cdot \Omega^{-1} \cdot \varepsilon_{t}\right) \\
& =\sum_{t=1}^{T} X_{t}^{\prime} \cdot \Omega^{-1} \cdot \frac{\partial\left(Z_{t}-X_{t} \cdot \beta\right)}{\partial \beta^{2}} \\
& =\sum_{t=1}^{T} X_{t}^{\prime} \cdot \Omega^{-1} \cdot X_{t} \\
& =-X^{\prime} \cdot \Sigma^{-1} \cdot X .
\end{aligned}
$$

$$
\mathscr{H}_{3,24: 23,38}=\frac{\partial^{2} \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \beta \partial \sigma^{2}}=\sum_{t=1}^{T} X_{t}^{\prime} \cdot \frac{\partial \Omega^{-1}}{\partial \sigma^{2}} \cdot \varepsilon_{t}
$$

The generic element of the above matrix is a $k \times 1$ vector:

$$
-\sigma_{1}^{-4} \cdot \sum_{t=1}^{T} X_{1, t}^{\prime} \cdot \varepsilon_{i, t}
$$

Going on with the calculation:

$$
\begin{aligned}
& \mathscr{H}_{i, i \mid i \in[24,38]}=\frac{\partial^{2} \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial\left(\sigma_{i}^{2}\right)^{2}}=\frac{T}{2} \cdot \frac{1}{\sigma_{i}^{4}} \cdot\left(1-\frac{2}{T \cdot \sigma_{i}^{2}} \cdot \sum_{t=1}^{T} \varepsilon_{i, t}^{2}\right) . \\
& \begin{aligned}
& \mathscr{H}_{23+i, 23+j \mid i, j \in[1, n]}=\frac{\partial^{2} \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \sigma_{i}^{2} \partial \sigma_{j}^{2}}=0 \quad \forall i \neq j . \\
& \mathscr{H}_{1,24: 1,38}= \frac{\partial^{2} \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \rho_{1} \partial \sigma_{i}^{2}}=\frac{\partial}{\partial \sigma_{i}^{2}}\left(\sum_{t \in t_{1}}^{T_{1}} \varepsilon_{t}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}\right) \\
&= \frac{\partial}{\partial \sigma_{i}^{2}}\left(\sum_{t \in t_{1}}^{T_{1}} \operatorname{Tr}\left(\varepsilon_{t}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}\right)\right) \\
&=\frac{\partial}{\partial \sigma_{i}^{2}}\left(\operatorname{Tr}\left(\left(\sum_{t \in t_{1}}^{T_{1}} \Delta y_{t} \cdot \varepsilon_{t}^{\prime}\right) \cdot \Omega^{-1} \cdot W_{1}\right)\right) \\
&= \operatorname{Tr}\left(\left(\sum_{t \in t_{1}}^{T_{1}} \Delta y_{t} \cdot \varepsilon_{t}^{\prime}\right) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_{i}^{2}} \cdot W_{1}\right)
\end{aligned}
\end{aligned}
$$

Note that

$$
\frac{\partial \Omega^{-1}}{\partial \sigma_{i}^{2}}=\left[\begin{array}{ccccc}
0 & \cdots & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & & \vdots \\
0 & \cdots & -\sigma_{i}^{-4} & \cdots & 0 \\
\vdots & & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \cdots & 0
\end{array}\right]=\operatorname{diag}\left(0, \cdots, 0,-\sigma_{i}^{-4}, 0, \cdots, 0\right)
$$

Symmetrically:

$$
\mathscr{H}_{2,24: 2,38}=\frac{\partial^{2} \log \mathscr{L}(\theta \mid \Delta y, X)}{\partial \rho_{2} \partial \sigma_{i}^{2}}=\operatorname{Tr}\left(\left(\sum_{t \in t_{2}}^{T_{2}} \Delta y_{t} \cdot \varepsilon_{t}^{\prime}\right) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_{i}^{2}} \cdot W_{2}\right)
$$

At this point we have all the elements to construct the Hessian matrix of the log-likelihood.
To sum up, first row:

- $\mathscr{H}_{1,1}=-T_{1} \cdot \operatorname{Tr}\left(W_{1} \cdot H_{\tau} \cdot W_{1} \cdot H_{\tau}\right)-\sum_{t \in t_{1}}^{T_{1}}\left(\Delta y_{t}^{\prime} \cdot W_{1}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}\right)$
- $\mathscr{H}_{1,2}=0$
- $\mathscr{H}_{1,3: 1,23}=-\sum_{t \in t_{1}}^{T_{1}} X_{t}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}$
- $\mathscr{H}_{1,24: 1,38}=\operatorname{Tr}\left(\left(\sum_{t \in t_{1}}^{T_{1}} \Delta y_{t} \cdot \varepsilon_{t}^{\prime}\right) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_{i}^{2}} \cdot W_{1}\right)$.

Second row:

- $\mathscr{H}_{2,1}=0$
- $\mathscr{H}_{2,2}=-T_{2} \cdot \operatorname{Tr}\left(W_{2} \cdot H_{\gamma} \cdot W_{2} \cdot H_{\gamma}\right)-\sum_{t \in t_{2}}^{T_{2}}\left(\Delta y_{t}^{\prime} \cdot W_{2}^{\prime} \cdot \Omega^{-1} \cdot W_{2} \cdot \Delta y_{t}\right)$
- $\mathscr{H}_{2,3: 2,23}=-\sum_{t \in t_{2}}^{T_{2}} X_{t}^{\prime} \cdot \Omega^{-1} \cdot W_{2} \cdot \Delta y_{t}$
- $\mathscr{H}_{2,24: 2,38}=\operatorname{Tr}\left(\left(\sum_{t \in t_{2}}^{T_{2}} \Delta y_{t} \cdot \varepsilon_{t}^{\prime}\right) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_{i}^{2}} \cdot W_{2}\right)$.

From row 3 to row 23:

- $\mathscr{H}_{3,1: 23,1}=\mathscr{H}_{1,3: 1,23}^{\prime}$
- $\mathscr{H}_{3,2: 23,2}=\mathscr{H}_{2,3: 2,23}^{\prime}$
- $\mathscr{H}_{3,3: 23,23}=\sum_{t=1}^{T} X_{t}^{\prime} \cdot \Omega^{-1} \cdot X_{t}$
- $\mathscr{H}_{3,24: 23,38}=\sum_{t=1}^{T} X_{t}^{\prime} \cdot \frac{\partial \Omega^{-1}}{\partial \sigma^{2}} \cdot \varepsilon_{t}$

From row 24 to the last row (number 38):

- $\mathscr{H}_{24,1: 38,1}=\mathscr{H}_{1,24: 1,38}^{\prime}$
- $\mathscr{H}_{24,2: 38,2}=\mathscr{H}_{2,24: 2,38}^{\prime}$
- $\mathscr{H}_{24,3: 38,23}=\mathscr{H}_{3,24: 23,38}^{\prime}$

$$
\text { - } \mathscr{H}_{23+i, 23+j \mid i, j \in[1, n]}=\left\{\begin{array}{l}
\frac{T}{2} \cdot \frac{1}{\sigma_{i}^{4}} \cdot\left(1-\frac{2}{T \cdot \sigma_{i}^{2}} \cdot \sum_{t=1}^{T} \varepsilon_{i, t}^{2}\right) \quad \forall i=j \in[1, n] \\
0 \quad \forall i \neq j
\end{array}\right.
$$

The last step we have to make to finally obtain the Fisher Information Matrix is taking expectations of every element.

$$
\begin{aligned}
E\left[\mathscr{H}_{1,1}\right] & =-T_{1} \cdot \operatorname{Tr}\left(W_{1} \cdot H_{\tau} \cdot W_{1} \cdot H_{\tau}\right)-\sum_{t \in t_{1}}^{T_{1}} E\left[\Delta y_{t}^{\prime} \cdot W_{1}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}\right]= \\
& =-T_{1} \cdot \operatorname{Tr}\left(W_{1} \cdot H_{\tau} \cdot W_{1} \cdot H_{\tau}\right)-\sum_{t \in t_{1}}^{T_{1}} E\left[\operatorname{Tr}\left(W_{1} \cdot \Delta y_{t} \cdot \Delta y_{t}^{\prime} \cdot W_{1}^{\prime} \cdot \Omega^{-1}\right)\right]= \\
& =-T_{1} \cdot \operatorname{Tr}\left(W_{1} \cdot H_{\tau} \cdot W_{1} \cdot H_{\tau}\right)-\sum_{t \in t_{1}}^{T_{1}} \operatorname{Tr}\left(W_{1} \cdot E\left[\Delta y_{t} \cdot \Delta y_{t}^{\prime}\right] \cdot W_{1}^{\prime} \cdot \Omega^{-1}\right)= \\
& =-T_{1} \cdot \operatorname{Tr}\left(W_{1} \cdot H_{\tau} \cdot W_{1} \cdot H_{\tau}\right)-\sum_{t \in t_{1}}^{T_{1}} \operatorname{Tr}\left(W _ { 1 } \cdot E \left[H_{\tau} \cdot X_{t} \cdot \beta \cdot \varepsilon_{t}^{\prime} \cdot H_{\tau}^{\prime}+\right.\right. \\
& \left.\left.+H_{\tau} \cdot X_{t} \cdot \beta \cdot \beta^{\prime} \cdot X_{t}^{\prime} \cdot H_{\tau}^{\prime}+H_{\tau} \cdot \varepsilon_{t} \cdot \varepsilon_{t}^{\prime} \cdot H_{\tau}^{\prime} \cdot \varepsilon_{t} \cdot \beta^{\prime} \cdot X_{t}^{\prime} \cdot H_{\tau}^{\prime}\right] \cdot W_{1}^{\prime} \cdot \Omega^{-1}\right)= \\
& =-T_{1} \cdot \operatorname{Tr}\left(W_{1} \cdot H_{\tau} \cdot W_{1} \cdot H_{\tau}\right)-\sum_{t \in t_{1}}^{T_{1}} \operatorname{Tr}\left(W _ { 1 } \cdot \left[H_{\tau} \cdot X_{t} \cdot \beta \cdot E\left[\varepsilon_{t}^{\prime}\right] \cdot H_{\tau}^{\prime}+\right.\right. \\
& \left.\left.+H_{\tau} \cdot X_{t} \cdot \beta \cdot \beta^{\prime} \cdot X_{t}^{\prime} \cdot H_{\tau}^{\prime}+H_{\tau} \cdot E\left[\varepsilon_{t} \cdot \varepsilon_{t}^{\prime}\right] \cdot H_{\tau}^{\prime}+E\left[\varepsilon_{t}\right] \cdot \beta^{\prime} \cdot X_{t}^{\prime} \cdot H_{\tau}^{\prime}\right] \cdot W_{1}^{\prime} \cdot \Omega^{-1}\right)= \\
& =-T_{1} \cdot \operatorname{Tr}\left(W_{1} \cdot H_{\tau} \cdot W_{1} \cdot H_{\tau}\right)- \\
& -\sum_{t \in t_{1}}^{T_{1}} \operatorname{Tr}\left(W_{1} \cdot\left[H_{\tau} \cdot X_{t} \cdot \beta \cdot \beta^{\prime} \cdot X_{t}^{\prime} \cdot H_{\tau}^{\prime}+H_{\tau} \cdot \Omega \cdot H_{\tau}^{\prime}\right] \cdot W_{1}^{\prime} \cdot \Omega^{-1}\right)= \\
& =-T_{1} \cdot \operatorname{Tr}\left(W_{1} \cdot H_{\tau} \cdot W_{1} \cdot H_{\tau}\right)- \\
& -\sum_{t \in t_{1}}^{T_{1}} \operatorname{Tr}\left(W_{1} \cdot H_{\tau} \cdot X_{t} \cdot \beta \cdot \beta^{\prime} \cdot X_{t}^{\prime} \cdot H_{\tau}^{\prime} \cdot W_{1}^{\prime} \cdot \Omega^{-1}+W_{1} \cdot H_{\tau} \cdot \Omega \cdot H_{\tau}^{\prime} \cdot W_{1}^{\prime} \cdot \Omega^{-1}\right)= \\
& =-T_{1} \cdot \operatorname{Tr}\left(W_{1} \cdot H_{\tau} \cdot W_{1} \cdot H_{\tau}+H_{\tau}^{\prime} \cdot W_{1}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot H_{\tau} \cdot \Omega\right)- \\
& -\sum_{t \in t_{1}}^{T_{1}} \operatorname{Tr}\left(\beta^{\prime} \cdot X_{t}^{\prime} \cdot H_{\tau}^{\prime} \cdot W_{1}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot H_{\tau} \cdot X_{t} \cdot \beta\right)=
\end{aligned}
$$

Setting $M_{1}^{\tau}=H_{\tau}^{\prime} \cdot W_{1}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot H_{\tau}$ we can rewrite the above identity as:

$$
\begin{aligned}
E\left[\mathscr{H}_{1,1}\right] & =-T_{1} \cdot \operatorname{Tr}\left(W_{1} \cdot H_{\tau} \cdot W_{1} \cdot H_{\tau}+M_{1}^{\tau} \cdot \Omega\right)-\sum_{t \in t_{1}}^{T_{1}} \beta^{\prime} \cdot X_{t}^{\prime} \cdot M_{1}^{\tau} \cdot X_{t} \cdot \beta= \\
& =-T_{1} \cdot \operatorname{Tr}\left(W_{1} \cdot H_{\tau} \cdot W_{1} \cdot H_{\tau}+M_{1}^{\tau} \cdot \Omega\right)-\beta^{\prime} \cdot X_{\tau}^{\prime} \cdot\left(I_{T_{1}} \otimes M_{1}^{\tau}\right) \cdot X_{\tau} \cdot \beta
\end{aligned}
$$

Simmetrically:
$E\left[\mathscr{H}_{2,2}\right]=-T_{2} \cdot \operatorname{Tr}\left(W_{2} \cdot H_{\gamma} \cdot W_{2} \cdot H_{\gamma}+M_{1}^{\gamma} \cdot \Omega\right)-\beta^{\prime} \cdot X_{\gamma}^{\prime} \cdot\left(I_{T_{2}} \otimes M_{1}^{\gamma}\right) \cdot X_{\gamma} \cdot \beta$. with $M_{1}^{\gamma}=H_{\gamma}^{\prime} \cdot W_{2}^{\prime} \cdot \Omega^{-1} \cdot W_{2} \cdot H_{\gamma}$.

Going on with the calculation:

$$
\begin{aligned}
E\left[\mathscr{H}_{1,3: 1,23}\right] & =E\left[-\sum_{t \in t_{1}}^{T_{1}} X_{t}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot \Delta y_{t}\right]= \\
& =-\sum_{t \in t_{1}}^{T_{1}} X_{t}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot E\left[H_{\tau} \cdot X_{t} \cdot \beta+H_{\tau} \cdot \varepsilon_{t}\right]= \\
& =-\sum_{t \in t_{1}}^{T_{1}} X_{t}^{\prime} \cdot \Omega^{-1} \cdot W_{1} \cdot H_{\tau} \cdot X_{t} \cdot \beta \\
& =X_{\tau}^{\prime} \cdot\left(I_{T_{1}} \otimes M_{2}^{\tau}\right) \cdot X_{\tau} \cdot \beta
\end{aligned}
$$

with $M_{2}^{\tau}=\Omega^{-1} \cdot W_{1} \cdot H_{\tau}$.

Simmetrically:

$$
E\left[\mathscr{H}_{2,3: 2,23}\right]=X_{\gamma}^{\prime} \cdot\left(I_{T_{2}} \otimes M_{2}^{\gamma}\right) \cdot X_{\gamma} \cdot \beta
$$

with $M_{2}^{\gamma}=\Omega^{-1} \cdot W_{2} \cdot H_{\gamma}$.

Next step:

$$
\begin{aligned}
E\left[\mathscr{H}_{1,24: 1,38}\right] & =\operatorname{Tr}\left(\left(\sum_{t \in t_{1}}^{T_{1}} E\left[\Delta y_{t} \cdot \varepsilon_{t}^{\prime}\right]\right) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_{i}^{2}} \cdot W_{1}\right)= \\
& =\operatorname{Tr}\left(\left(\sum_{t \in t_{1}}^{T_{1}} E\left[\Delta y_{t} \cdot \varepsilon_{t}^{\prime}\right]\right) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_{i}^{2}} \cdot W_{1}\right)= \\
& =\operatorname{Tr}\left(\left(\sum_{t \in t_{1}}^{T_{1}} H_{\tau} \cdot E\left[\varepsilon_{t} \cdot \varepsilon_{t}^{\prime}\right]\right) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_{i}^{2}} \cdot W_{1}\right)= \\
& =T_{1} \cdot \operatorname{Tr}\left(H_{\tau} \cdot \Omega \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_{i}^{2}} \cdot W_{1}\right)= \\
& =T_{1} \cdot \operatorname{Tr}\left(\Omega \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_{i}^{2}} \cdot W_{1} \cdot H_{\tau}\right)
\end{aligned}
$$

Notice that

$$
\Omega \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_{i}^{-2}}=-\sigma_{i}^{2} \cdot I_{i i}
$$

where the generic element of matrix $I_{i i}$ is given by

$$
\omega_{s, t}= \begin{cases}1 & s=i, j=i \\ 0 & \text { otherwise }\end{cases}
$$

Therefore

$$
\begin{aligned}
E\left[\mathscr{H}_{1,23+i}\right] & =T_{1} \cdot \sigma_{i}^{-2} \cdot \operatorname{Tr}\left(I_{i i} \cdot W_{1} \cdot H_{\tau}\right)= \\
& =T_{1} \cdot \sigma_{i}^{-2} \cdot\left(W_{1} \cdot H_{\tau}\right)_{i i}
\end{aligned}
$$

Finally we have that:

$$
E\left[\mathscr{H}_{1,24: 1: 38}\right]=T_{1} \cdot \operatorname{diag}\left(\Omega^{-1} \cdot W_{1} \cdot H_{\tau}\right)=T_{1} \cdot \operatorname{diag}\left(M_{2}^{\tau}\right) .
$$

Simmetrically:

$$
E\left[\mathscr{H}_{2,24: 2: 38}\right]=T_{2} \cdot \operatorname{diag}\left(\Omega^{-1} \cdot W_{2} \cdot H_{\gamma}\right)=T_{2} \cdot \operatorname{diag}\left(M_{2}^{\gamma}\right) .
$$

Going on:

$$
\begin{aligned}
& E\left[\mathscr{H}_{3,3: 23,23}\right]=E\left[\sum_{t=1}^{T} X_{t}^{\prime} \cdot \Omega^{-1} \cdot X_{t}\right]=\sum_{t=1}^{T} X_{t}^{\prime} \cdot \Omega^{-1} \cdot X_{t}=X^{\prime} \cdot \Sigma^{-1} \cdot X \\
& E\left[\mathscr{H}_{3,24: 23,38}\right]=E\left[\sum_{t=1}^{T} X_{t}^{\prime} \cdot \frac{\partial \Omega^{-1}}{\partial \sigma^{2}} \cdot \varepsilon_{t}\right] \\
& =\sum_{t=1}^{T} X_{t}^{\prime} \cdot \frac{\partial \Omega^{-1}}{\partial \sigma^{2}} \cdot E\left[\varepsilon_{t}\right] \\
& =\underset{k \times n}{0} \\
& E\left[\mathscr{H}_{23+i, 23+j \mid i, j \in[1, n]}\right]=\left\{\begin{array}{l}
\frac{T}{2} \cdot \frac{1}{\sigma_{i}^{4}} \cdot\left(1-\frac{2}{T \cdot \sigma_{i}^{2}} \cdot \sum_{t=1}^{T} E\left[\varepsilon_{i, t}^{2}\right]\right) \quad \forall i=j \in[1, n] \\
0 \quad \forall i \neq j
\end{array}\right. \\
& =\left\{\begin{array}{l}
-\frac{T}{2} \cdot \frac{1}{\sigma_{i}^{4}} \quad \forall i=j \in[1, n] \\
0 \quad \forall i \neq j
\end{array}\right. \\
& =-\frac{T}{2} \cdot\left[\begin{array}{cccc}
\sigma_{1}^{-4} & 0 & \cdots & 0 \\
0 & \sigma_{2}^{-4} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
o & 0 & \cdots & \sigma_{n}^{-4}
\end{array}\right]=-\frac{T}{2} \cdot V
\end{aligned}
$$

We finally have all the elements of the Fisher Information Matrix for our panel (with dummy variables) spatial model:

$$
\mathscr{I}=
$$

水

## C. 3 Bayesian MCMC - Technical Details

Even if the MLE is a common standard method in spatial econometric applications, we have two valid reasons for not adopting it: 1. non-stationary estimates of aggregate total effects; 2. prior information on the values of the parameters. Let's explore both the issues.

## 1. Non-Stationary Solutions

We can estimate the parameters by maximizing the concentrated log-likelihood over the compact set which guarantees a positive definite matrix (see Ord (1975)): $C^{\text {down }}=\left(\lambda_{\min }^{-1}, \lambda_{\max }^{-1}\right)$ and $C^{u p}=\left(\mu_{\min }^{-1}, \mu_{\max }^{-1}\right)$. The standard errors are constructed using the analytical Fisher Information of the model, centered on the point estimates, $\hat{\rho}^{\text {down }}$ and $\hat{\rho}^{u p}$. The asymptotic results of Yu, DeJong, and Lee (2008) guarantees the asymptotic normality of the parameters of equation (6) and (7) (See Theorem 3 case $n / T \rightarrow 0$ ). For instance, for the estimator of $\rho^{\text {down }}$ we have:

$$
\sqrt{T \cdot n}\left(\hat{\rho}_{n T}^{\text {down }}-\rho^{\text {down }}\right) \xrightarrow{d} \mathscr{N}\left(0, \sigma^{2}\right)
$$

where $\sigma^{2}$ is the asymptotic variance of the MLE, obtained by the calculating the analytical Fisher Information matrix of our model. However, we are interested in estimating the aggregate total effect of fiscal consolidations, not the parameters of the model themselves. At page 70, LeSage and Pace (2009) suggest to construct the asymptotic distribution of the average total effect (our aggregate total effect) by following these steps: 1. estimate the parameters of the model via MLE; 2. Draw values of the parameters by their approximate asymptotic distribution $\left(\tilde{\rho}^{\text {down }} \approx \mathscr{N}\left(\hat{\rho}_{n T}^{\text {down }}, \frac{\hat{\sigma}^{2}\left(\hat{\rho}_{n T}^{\text {dow }}\right)}{n T}\right) ; 3\right.$. Calculate at each step the aggregate total effect. After doing so we calculated the standard errors of the $A T E_{T B}$ by calculating the standard deviation of the asymptotic distribution so constructed. We obtained explosive solutions. This is a surprising result, in fact, the asymptotic normality of the average effect is guaranteed by the $\Delta$-method:

$$
\sqrt{T \cdot n}\left(A T E_{T B}\left(\hat{\rho}_{n T}^{d o w n}\right)-A T E_{T B}\left(\rho^{\text {down }}\right)\right) \xrightarrow{d} \mathscr{N}\left(0, \sigma^{2} \cdot\left(\frac{\partial A T E_{T B}\left(\rho^{d o w n}\right)}{\partial \rho^{d o w n}}\right)^{2}\right)
$$

where $A T E_{T B}: C^{\text {down }} \rightarrow \mathbb{R}$ and $A T E_{T B}(x)=v^{\prime} \cdot\left(I_{n}-x \cdot A\right)^{-1} \cdot \omega_{T B}$ and $v$ is a vector of industry output shares of total industrial production (the weights we use to calculate the aggregate effect of fiscal consolidations). What goes wrong in this procedure? The $\Delta$-method is an asymptotic result, which
might provide a terrible approximation of a finite sample distribution. It all boils down in finding a distribution which approximates well the small sample one. If $\hat{\rho}_{n T}^{\text {down }}$ is very closed to the boundary and its asymptotically normal standard errors are large, that is, they approach the boundary of $C^{d o w n}$ then we end up drawing values of $\rho^{\text {down }}$ which deliver unrealistically large values of $A T E_{T B}$, because matrix $\left(I_{n}-\rho^{\text {down }} \cdot A\right)^{-1}$ becomes singular (the boundary is one eigenvalue of $A$ ). This situation is described in Figure 13.

Figure 13: Explosive Solutions of $A T E_{T B}$


## 2. Prior Information

We have two extra "prior" pieces of information on the value of the spatial parameters, $\rho^{\text {down }}$ and $\rho^{u p}$ :
i. Values of $\rho^{\text {down }}$ and $\rho^{u p}$ close to the boundaries will deliver unrealistically high values of ATE, ADE and ANE, since the determinant of matrices $\left(I_{n}-\rho^{\text {down }} \cdot A\right)$ and $\left(I_{n}-\rho^{u p} \cdot \hat{A}^{T}\right)$ will approach zero by definition of eigenvalue. In turn, the elements of their inverse matrices will explode, as illustrated above. Therefore, we should assign less weight to values of $\rho^{\text {down }}$ and $\rho^{u p}$ close to the boundaries.
ii. We know that industries that are close to each other in the production network will co-move. For instance, if industry X faces increasing prices
for its input, it will shrink production and increase prices; in turn, customers of X will also face the same problem and will react similarly, by reducing production and increasing prices. Therefore, the direction of the spatial correlation among industries' output is positive: $\rho^{\text {down }}>0$ and $\rho^{u p}>0$.

## Model Estimation

We can integrate such prior information into our estimation and avoid nonstationarity aggregate effects, by adopting a Bayesian MCMC similar to the one introduced by LeSage and Parent (2007). We illustrate here how we implement the Bayesian MCMC to estimate the parameters of Equation (6) (baseline). The log-likelihood of that model is the one outlined above. The priors we employ on the parameters are:

$$
\begin{aligned}
& \pi(\beta) \propto \text { constant } \\
& \Omega=\sigma^{2} \cdot V \quad \text { with } V=\operatorname{diag}\left(v_{1}, \ldots, v_{n}\right) \\
& \pi\left(\sigma^{2}\right) \propto \frac{1}{\sigma^{2}} \\
& \pi\left(v_{i}\right) \stackrel{i i d}{\sim} \Gamma^{-1}\left(\frac{r}{2}, \frac{r}{2}\right), \quad i=1, \ldots, n \\
& \rho^{d o w n} \sim \operatorname{Gen.Beta}(d, d) \\
& \rho^{u p} \sim \operatorname{Gen.Beta}(d, d) .
\end{aligned}
$$

We adopt non-informative priors for $\sigma^{2}$ and $\beta$ to reflect our lack of information around the values of these parameters. Concerning $r$, a lower value generates more diffusion in the distributions of $v_{i}$, thus regulating our confidence towards heteroskedasticity. Unlike LeSage and Pace (2009), who suggest a value of 4, we set $r$ equal to 3 to reflect a strong belief towards heteroskedasticity. For instance, industries in the Agriculture (NAICS 11) as well as Mining (NAICS 21) macro sectors, exhibit much higher volatilities than the rest of the industries.
We impose a "generalized (or non-standardized) $\operatorname{Beta}(d, d)$ prior", with support from 0 to $\lambda_{\text {max }}^{-1}$ for $\rho^{\text {down }}$ and from 0 to $\hat{\lambda}_{\text {max }}^{-1}$ for $\rho^{u p}$. We follow LeSage and Pace (2009) and set $d$ equal to 1.1; which has the benefit of letting the generalized Beta prior to resemble a Uniform distribution (diffuse prior), but with low density at the boundaries, as illustrated in Figure 14. The choice of such a prior allows us to be agnostic about the specific value of the spatial parameters but at the same time it allows to embed the prior information we

Figure 14: Generalized Beta prior


Figure 14: line-plot of a non-standardized Beta(1.1,1.1) density function, with support from $\left(0, \lambda_{\max }^{-1}(A)=2.047\right)$ which we employ as a prior for the spatial parameter $\rho^{\text {down }}$.
have into their estimates.
Furthermore, we assume that all the prior distributions are independent from each other. We use the standard "Metropolis within Gibbs" algorithm, and we obtain an approximation of the posterior densities for each parameter of the model.
We now outline the precise steps of the procedure:

1. Initialization: Set up initial values for the parameters: $\beta_{(0)}, \sigma_{(0)}^{2}, V_{(0)}, \rho_{(0)}^{d o w n}, \rho_{(0)}^{u p}$, where $V_{(0)}=\operatorname{diag}\left(v_{1,(0)}^{2}, \ldots, v_{n,(0)}^{2}\right)$.
2. Gibbs Sampling:
a) Draw $\beta_{(1)}$ from the conditional posterior distribution, which is obtained by mixing the likelihood with a normal prior with mean $c$ (a vector of zeros in our simulation) and covariance matrix $L$. In order to not add any information, we simply set $L$ to be equal to a
diagonal matrix whose entries are infinite (1e12 in our simulation):

$$
\begin{aligned}
& P\left(\beta_{(0)} \mid \mathscr{D}, \sigma_{(0)}^{2}, V_{(0)}, \rho_{(0)}^{\text {down }}, \rho_{(0)}^{u p}\right)=\mathscr{N}\left(c^{*}, L^{*}\right) \propto \mathscr{L}(\theta \mid \mathscr{D}) \cdot \mathscr{N}(c, L) \\
& c^{*}=\frac{1}{T} \cdot\left(\sum_{t=1}^{T} X_{t}^{\prime} \cdot V_{(0)}^{-1} \cdot X_{t}+\frac{\sigma_{(0)}^{2}}{T} \cdot L^{-1}\right)^{-1} \cdot\left(\frac{1}{T} \cdot \sum_{t=1}^{T} X_{t}^{\prime} \cdot V_{(0)}^{-1} \cdot H_{t} \cdot \Delta y_{t}+\frac{\sigma_{(0)}^{2}}{T} \cdot L^{-1} \cdot c\right) \\
& L^{*}=\frac{\sigma_{(0)}^{2}}{T} \cdot\left(\sum_{t=1}^{T} X_{t}^{\prime} \cdot V_{(0)}^{-1} \cdot X_{t}+\frac{\sigma_{(0)}^{2}}{T} \cdot L^{-1}\right)^{-1}
\end{aligned}
$$

b) Draw $\sigma_{(1)}^{2}$ from the conditional posterior distribution, which is proportional to likelihood times an inverse gamma distribution as a prior:

$$
\begin{aligned}
& P\left(\sigma_{(1)}^{2} \mid \mathscr{D}, \beta_{(1)}, V_{(0)}, \rho_{(0)}^{d o w n}, \rho_{(0)}^{u p}\right)=\Gamma^{-1}\left(\frac{\theta_{1}}{2}, \frac{\theta_{2}}{2}\right) \propto \mathscr{L}(\theta \mid \mathscr{D}) \cdot \Gamma^{-1}(a, b) \\
& \theta_{1}=n T+2 a \quad \theta_{2}=\sum_{t=1}^{T} \varepsilon_{t}^{\prime} \cdot V_{(0)}^{-1} \cdot \varepsilon_{t}+2 b
\end{aligned}
$$

In practice we draw $\sigma_{(1)}^{2}$ from $\theta_{2} / \chi_{\theta_{1}}$.
Notice that, setting $a$ and $b$ (the prior's parameters) equal to 0 , is like putting a Jefferey's prior on $\sigma^{2}$. This is exactly what we do.
c) Draw $v_{i,(1)}$ from the following conditional posterior distribution, proportional to an inverse gamma prior:

$$
\begin{aligned}
& P\left(v_{i,(1)} \mid \mathscr{D}, \sigma_{(1)}^{2}, \rho_{(0)}^{d o w n}, \rho_{(0)}^{u p}\right)=\Gamma^{-1}\left(\frac{q_{1}}{2}, \frac{q_{2}}{2}\right) \propto \mathscr{L}(\theta \mid \mathscr{D}) \cdot \Gamma^{-1}\left(\frac{r}{2}, \frac{r}{2}\right) \\
& q_{1}=r+T \quad q_{2}=\frac{1}{\sigma_{(1)}^{2}} \cdot \sum_{t=1}^{T} \varepsilon_{i, t}^{2}+r
\end{aligned}
$$

In practice we draw $v_{i,(1)}$ from $q_{2} / \chi_{q_{1}}$.
As anticipated above in the paper, since we are confident on the heteroskedastic behavior of industry value added, we set our prior hyperparameter $r$ to be equal to 3 rather than 4, as done in LeSage and Pace (2009).
Replicating this procedure $n$ times, we get a first simulation of matrix $V_{(1)}$.
3. Metropolis-Hastings: We now need to draw the spatial coefficients. However we cannot apply a simple Gibbs Sampling, since the conditional posterior distribution is not defined for them. LeSage and Pace (2009)
suggest the adoption of the Metropolis-Hastings algorithm to overcome this problem. To ease notation we set $\rho_{1}:=\rho^{\text {down }}$ and $\rho_{2}:=\rho^{u p}$. The algorithm is the following:
(a) Draw $\rho_{1}^{c}$ (where the $c$ superscript stands for "candidate") from the (random walk) proposal distribution:

$$
\rho_{1}^{c}=\rho_{1,(0)}+c_{1} \cdot \mathscr{N}(0,1)
$$

(b) Run a bernoulli experiment to determine the updated value of $\rho_{1}$ :

$$
\rho_{1,(1)}=\left\{\begin{array}{lll}
\rho_{1}^{c} & \pi & (\text { accept }) \\
\rho_{1,(0)} & 1-\pi & (\text { reject })
\end{array}\right.
$$

Where $\pi$ is equal to $\pi=\min \left\{1, \psi_{M H_{1}}\right\}$ and, setting: $A_{\tau}\left(\rho_{1}\right)=$ $I_{n}-\rho_{1} \cdot W_{1}$, we have:

$$
\begin{aligned}
\psi_{M H_{1}} & =\frac{\left|A_{\tau}\left(\rho_{1}^{c}\right)\right|}{\left|A_{\tau}\left(\rho_{1,(0)}\right)\right|} \cdot \exp \left\{-\frac{1}{2 \sigma_{(1)}^{2}} \cdot \sum_{t \in t_{1}}^{T_{1}}\left[\Delta y _ { t } ^ { \prime } \cdot \left(A_{\tau}\left(\rho_{1}^{c}\right)^{\prime} \cdot V_{(1)}^{-1} \cdot A_{\tau}\left(\rho_{1}^{c}\right)-\right.\right.\right. \\
& \left.-A_{\tau}\left(\rho_{1,(0)}\right)^{\prime} \cdot V_{(1)}^{-1} \cdot A_{\tau}\left(\rho_{1,(0)}\right)\right) \cdot \Delta y_{t}- \\
& \left.\left.-2 \beta^{\prime} \cdot X_{t}^{\prime} \cdot V_{(1)}^{-1}\left(A_{\tau}\left(\rho_{1}^{c}\right)-A_{\tau}\left(\rho_{1,(0)}\right)\right) \cdot \Delta y_{t}\right]\right\} . \\
& \cdot\left[\frac{\left(\rho_{1}^{c}-0\right) \cdot\left(\lambda_{\max }^{-1}-\rho_{1}^{c}\right)}{\left(\rho_{1,(0)}-0\right) \cdot\left(\lambda_{\max }^{-1}-\rho_{1,(0)}\right)}\right]^{d-1} \cdot \mathbf{1}\left(0 \leq \rho_{1}^{c} \leq \lambda_{\max }^{-1}\right)
\end{aligned}
$$

Basically, we compute the probability to accept the candidate value from the proposal distribution, and then we update the value of $\rho_{1}$ by running the bernoulli experiment with such a probability of success. Notice that if we draw a value of $\rho_{1}$ outside the support of the beta prior, $\psi_{M H_{1}}=0$ and then $\pi=0$ and we clearly reject the candidate value.
We set $d$ equal to 1.1 , on both $\rho_{1}$ and $\rho_{2}$; this is done to resemble a Uniform $(0,1)$ but with less density on its boundary values.
(c) Once updated $\rho_{1}$, we replicate the procedure for $\rho_{2}$. Setting $A_{\gamma}\left(\rho_{2}\right)=$
$I_{n}-\rho_{2} \cdot W_{2}$ we have:

$$
\begin{aligned}
\psi_{M H_{2}} & =\frac{\left|A_{\gamma}\left(\rho_{2}^{c}\right)\right|}{\left|A_{\gamma}\left(\rho_{2,(0)}\right)\right|} \cdot \exp \left\{-\frac{1}{2 \sigma_{(1)}^{2}} \cdot \sum_{t \in t_{2}}^{T_{2}}\left[\Delta y _ { t } ^ { \prime } \cdot \left(A_{\gamma}\left(\rho_{2}^{c}\right)^{\prime} \cdot V_{(1)}^{-1} \cdot A_{\gamma}\left(\rho_{2}^{c}\right)-\right.\right.\right. \\
& \left.-A_{\gamma}\left(\rho_{2,(0)}\right)^{\prime} \cdot V_{(1)}^{-1} \cdot A_{\gamma}\left(\rho_{2,(0)}\right)\right) \cdot \Delta y_{t}- \\
& \left.\left.-2 \beta^{\prime} \cdot X_{t}^{\prime} \cdot V_{(1)}^{-1}\left(A_{\gamma}\left(\rho_{2}^{c}\right)-A_{\gamma}\left(\rho_{2,(0)}\right)\right) \cdot \Delta y_{t}\right]\right\} . \\
& \cdot\left[\frac{\left(\rho_{2}^{c}-0\right) \cdot\left(\hat{\lambda}_{\max }^{-1}-\rho_{2}^{c}\right)}{\left(\rho_{2,(0)}-0\right) \cdot\left(\hat{\lambda}_{\max }^{-1}-\rho_{2,(0)}\right)}\right]^{d-1} \cdot \mathbf{1}\left(0 \leq \rho_{2}^{c} \leq \hat{\lambda}_{\max }^{-1}\right)
\end{aligned}
$$

(d) At this point we need to update the variance of the proposal distributions: if the acceptance rate (number of acceptances over number of iterations of the Markov Chain) of the first parameter $\rho_{1}$ falls below $40 \%$ we need to reduce the value of $c_{1}$, the so called tuning parameter, which regulates the variance of the proposal distribution. The variance is reduced by rescaling it: $c_{1}^{\prime}=\frac{c_{1}}{1.1}$. In this way, we are able to draw values closer to the current state of $\rho_{1}$, and therefore, we expect to increase the acceptance rate.
On the contrary, if the acceptance rate rises above $60 \%$, we need to increase the tuning parameter, in order to draw values far from the current state, in this way we increase the chance to explore more the low-density parts of the distribution. We increase the variance of the candidate distribution by scaling upward its standar deviation: $c_{1}^{\prime}=1.1 \cdot c_{1}$.
Clearly we replicate this procedure also for $\rho_{2}$.
4. Repeat: Once updated all the values, we replicate steps 2 and 3, 45,000 times to make sure the acceptance rate has converged.
5. Burn-in: we drop the first 35,000 iterations of the Markov Chain, thus obtaining a vector of 10,000 observations for each of the parameters, which account for the simulated posterior distributions.

## C. 4 Simulating the ATE, ADE and ANE

We construct via Monte Carlo the distribution of the ATE, ADE and ANE. In particular we follow these steps:

1. (Parameters) Draw $\rho^{\text {down }}, \rho^{u p}, \boldsymbol{\tau}$ and $\boldsymbol{\gamma}$ from their posterior distributions. To take into account the potential correlation among them, draw from the same iteration of the Bayesian MCMC.
2. (Style of the plan) Construct both a TB and an EB simulated fiscal plan, by drawing the style from a distribution which mimics the empirical one.
3. (Average effects) Construct ATE, ADE and ANE using the parameters drawn in step 1 and the style drawn in step 2.
4. Repeat 100,000 times steps from 1 though 3 , to make sure all the possible combination of styles and parameters are simulated.

Step 2 allows us to claim that the baseline results reported in the paper are robust to different styles of fiscal plans.

## Empirical distribution of style of fiscal plans

We are interested in simulating a 2 years fiscal consolidation made of an unexpected part, no announced part and a single year future part to be implemented in the second year of the simulation.
First of all, we want to simulate the unexpected part of the fiscal plan, therefore, we need to look at those years when an unanticipated shock occurs. Define the two sub-samples: $T B^{u}:=\left\{t: 1, \ldots, T \mid t a x_{t}^{u}>0\right\}$ and $E B^{u}:=\left\{t: 1, \ldots, T \mid \exp _{t}^{u}>0\right\}$. Then calculate the mean and the standard deviation of the unexpected component conditional on the occurrence of an unexpected shock:

$$
\begin{array}{ll}
\mu_{\tau}:=\mathbb{E}\left(\operatorname{tax}_{t}^{u} \mid t \in T B^{u}\right) & \sigma_{\tau}:=\sqrt{\mathbb{V}\left(\operatorname{tax}_{t}^{u} \mid t \in T B^{u}\right)} \\
\mu_{\gamma}:=\mathbb{E}\left(\exp _{t}^{u} \mid t \in E B^{u}\right) & \sigma_{\gamma}:=\sqrt{\mathbb{V}\left(\exp _{t}^{u} \mid t \in E B^{u}\right)}
\end{array}
$$

In order to simulate a plausible unexpected component of the plan, we draw them from the following distributions:

$$
\begin{aligned}
& t \tilde{a x} x^{u} \sim \mathscr{U}\left(\mu_{\tau}-\sigma_{\tau}, \mu_{\tau}+\sigma_{\tau}\right) \\
& e \tilde{x} p^{u} \sim \mathscr{U}\left(\mu_{\gamma}-\sigma_{\gamma}, \mu_{\gamma}+\sigma_{\gamma}\right)
\end{aligned}
$$

where the ${ }^{\sim}$ denotes a simulated component.
Concerning the future component, we need to predict what is the value of a one year ahead policy change, conditional on the occurrence of an unexpected policy change. Therefore, we run the following regressions:

$$
\begin{array}{ll}
t a x_{t, 1}^{f}=a_{\tau}+b_{\tau} \cdot t a x_{t}^{u} & \text { with }: t \in T B^{u} \\
\exp _{t, 1}^{f}=a_{\gamma}+b_{\gamma} \cdot \exp _{t}^{u} & \text { with: } t \in E B^{u}
\end{array}
$$

The estimates of $a_{\tau}, b_{\tau}, a_{\gamma}, b_{\gamma}$ will be stored and used to predict values of tax $x_{t, 1}^{f}$ and $\exp _{t, 1}^{f}$, conditional on the occurrence of an unexpected component. At this point we have all the ingredients to outline the steps we do in the construction of a simulated style of the plan:

1. Draw unexpected components from their candidate distributions: $t \tilde{a} x{ }^{u} \sim$ $\mathscr{U}\left(\mu_{\tau}-\sigma_{\tau}, \mu_{\tau}+\sigma_{\tau}\right)$ and $e \tilde{x} p^{u} \sim \mathscr{U}\left(\mu_{\gamma}-\sigma_{\gamma}, \mu_{\gamma}+\sigma_{\gamma}\right)$.
2. Predict the future component using the estimates of $a_{\tau}, b_{\tau}, a_{\gamma}, b_{\gamma}$. We have: $\operatorname{ta} x^{f}=\hat{a}_{\tau}+\hat{b}_{\tau} \cdot t \tilde{a} x^{u}$ and $e \tilde{x} p^{f}=\hat{a}_{\gamma}+\hat{b}_{\gamma} \cdot e \tilde{x} p^{u}$.
3. Normalize the value to one: $t \tilde{a} x^{u}+t \tilde{a} x^{f}=1$ and $e \tilde{x} p^{u}+e \tilde{x} p^{f}=1$.

For each iteration of the MC simulation used to approximate the posterior distributions of the ATE, ADE and ANE, we repeat steps 1 through 3 to simulate the style of the plan.
In the first year of the simulation we calculate the effects of TB and EB plans with style given by: $\boldsymbol{s}_{T B}=\left[t \tilde{a} x^{u} 0 t \tilde{a} x^{f}\right]$ and $\boldsymbol{s}_{E B}=\left[e \tilde{x} p^{u} 0 e \tilde{x} p^{f}\right]$ respectively. In the second year of the simulation, the future component of the shock is rolled over and becomes an announced and implemented shock. Therefore we calculate the effects of TB and EB plans with style given by: $\boldsymbol{s}_{T B}=\left[\begin{array}{ll}0 & \tilde{a} x\end{array}{ }^{f} 0\right]$ and $\boldsymbol{s}_{E B}=\left[\begin{array}{ll}0 & e \tilde{x} p^{f}\end{array} 0\right]$ respectively.

## D Estimates of Inverted Model

In this section we report the tables of estimates of the inverted model. Firstly, Table VII shows the estimates of Equation (7).

## D. 1 Model Selection - Vuong Test for Static Spatial Panel Data

We also provide results for a Vuong test of non-nested models, adapted to our spatial specification, as in Wooldridge (2010).

Firstly, the Vuong test (see Vuong (1989)) is meant to discriminate between two misspecified and non-nested models. Basically, we assume there is a hidden true model and we want to choose one of two competing nonnested models which fit the data equally well. The Vuong test calculates and compares the Kullback-Leibler distance between the two and the true model. In practice, is a t-test on the KL divergence. One problem we encounter is that it was developed for one-dimensional iid data, however, we deal with

Table VII: Estimation Results

| Inverted Model - Equation (7) |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Parameters | MLE |  | Bayesian MCMC - Posterior Distributions: |  |  |  |  |  |  |  |  |  |
|  | $\hat{\theta}_{i}^{\text {ML }}$ | MLE Std. | $\mathbb{E}\left(\theta_{i}\right)$ | $\sqrt{\mathbb{V}\left(\theta_{i}\right)}$ | $\operatorname{Pr}\left(\theta_{i}<0\right)$ | $5 \%$ | 10\% | 16\% | 50\% | 84\% | 90\% | 95\% |
| $\rho^{u p}$ (TB) | 0.554 | 0.103 | 0.528 | 0.097 | 0.000 | 0.368 | 0.405 | 0.432 | 0.528 | 0.625 | 0.653 | 0.687 |
| $\tau_{u}$ | 0.684 | 1.283 | 0.815 | 1.193 | 0.247 | -1.143 | -0.712 | -0.372 | 0.814 | 2.002 | 2.351 | 2.778 |
| $\tau_{a}$ | -1.298 | 0.986 | -1.290 | 0.919 | 0.920 | -2.794 | -2.463 | -2.202 | -1.293 | -0.382 | -0.112 | 0.225 |
| $\tau_{f}$ | -0.080 | 0.426 | -0.084 | 0.391 | 0.585 | -0.726 | -0.585 | -0.474 | -0.082 | 0.301 | 0.415 | 0.562 |
| $\rho^{\text {down }}(\mathrm{EB})$ | 0.096 | 0.114 | 0.125 | 0.083 | 0.000 | 0.014 | 0.026 | 0.040 | 0.112 | 0.211 | 0.241 | 0.281 |
| $\gamma_{u}$ | 0.073 | 1.126 | 0.050 | 1.034 | 0.480 | -1.650 | -1.272 | -0.973 | 0.051 | 1.073 | 1.370 | 1.760 |
| $\gamma_{a}$ | 1.286 | 0.617 | 1.296 | 0.567 | 0.011 | 0.361 | 0.572 | 0.732 | 1.295 | 1.861 | 2.023 | 2.226 |
| $\gamma_{f}$ | -0.502 | 0.282 | -0.499 | 0.259 | 0.973 | -0.923 | -0.831 | -0.757 | -0.499 | -0.241 | -0.169 | -0.075 |
| D2008 | -2.984 | 0.674 | -2.934 | 0.633 | 1.000 | -3.973 | -3.744 | -3.562 | -2.936 | -2.307 | -2.120 | -1.891 |
| D2009 | -5.710 | 0.674 | -5.371 | 0.661 | 1.000 | -6.469 | -6.216 | -6.025 | -5.368 | -4.717 | -4.529 | -4.290 |

Table VII: $\theta_{i}$ denotes a generic parameter that we estimate. The columns report the following: $\hat{\theta}_{i}^{M L}$ is the ML point estimate; "MLE Std." is the standard deviation of the ML estimate, calculated using the analytical Fisher Information Matrix derived in Appendix C.2: $\sqrt{\mathscr{I}\left(\hat{\theta}^{M L}\right)_{i i}^{-1}} ; \mathbb{E}\left(\theta_{i}\right)$ is the expected value of the posterior distribution; $\sqrt{\mathbb{V}\left(\theta_{i}\right)}$ is the standard deviation of the posterior distribution; $\operatorname{Pr}(\theta<0)$ is the probability that a parameter is negative, calculated by integrating the posterior distribution; $p \%$ is the $p$-th percentile of the posterior distribution. For brevity we don't report here the Industry Fixed Effects and the Industry specific variances. In the first columns, the spatial parameters also report the type of fiscal plan they are interacted with (in blue).
a panel whose observations are serially uncorrelated but spatially correlated. Wooldridge (2010) shows that the Vuong test can easily be extended to panel data models by accounting for serial correlation in the time series. ${ }^{38}$ However, in our problem the $n \times 1$ vector of industry observations is iid over time and our asymptotic keeps the cross-sectional dimension, which is spatially correlated, fixed, and then let the time series to go to infinite $T \rightarrow \infty$. Economically speaking this makes sense: we observe those fixed 62 industries over time, however, the cross sectional dimension exceeds the times series one, 37 years. This means that our finite sample distribution will not be a very good approximation of the asymptotic one. However, this is the best we can do, given the data availability.
Let's derive now the Vuong Test. The quasi-log-likelihood of the baseline model, Equation (6), is:

$$
\begin{aligned}
& \ell_{t, B}(\underbrace{\rho, \beta, \Omega}_{\theta_{B}})=\log f_{B}\left(\Delta y_{t} \mid X_{t} ; \theta_{B}\right)=-\frac{n}{2} \ln (2 \pi)-\frac{1}{2} \cdot \ln (|\Omega|)+ \\
& \quad+\ln \left(\left|I_{n}-\rho^{\text {down }} \cdot A \cdot T B_{t}-\rho^{u p} \cdot \hat{A}^{\prime} \cdot E B_{t}\right|\right)-\frac{1}{2} \cdot \varepsilon_{t}^{\prime} \cdot \Omega^{-1} \cdot \varepsilon_{t} .
\end{aligned}
$$

with:

$$
\varepsilon_{t}=\left(I_{n}-\rho^{d o w n} A T B_{t}-\rho^{u p} \hat{A}^{\prime} E B_{t}\right) \cdot \Delta y_{t}-X_{t} \cdot \beta
$$

[^25]The sum of the quasi-log-likelihood evaluated at the MLE, $\hat{\theta}_{B}$, for the baseline model is: $\mathscr{L}_{B}=\sum_{t=1}^{T} \ell_{t, B}\left(\hat{\theta}_{B}\right)$. Analogously, for the inverted model, Equation (7), the quasi-log-likelihood is:

$$
\begin{aligned}
& \ell_{t, I}(\underbrace{\tilde{\rho}, \tilde{\beta}, \tilde{\Omega}}_{\theta_{I}})=\log f_{I}\left(\Delta y_{t} \mid X_{t} ; \theta_{I}\right)=-\frac{n}{2} \ln (2 \pi)-\frac{1}{2} \cdot \ln (|\tilde{\Omega}|)+ \\
& \quad+\ln \left(\left|I_{n}-\tilde{\rho}^{\text {down }} \cdot A \cdot E B_{t}-\tilde{\rho}^{\text {up }} \cdot \hat{A}^{\prime} \cdot T B_{t}\right|\right)-\frac{1}{2} \cdot \varepsilon_{t}^{\prime} \cdot \tilde{\Omega}^{-1} \cdot \varepsilon_{t} .
\end{aligned}
$$

with:

$$
\varepsilon_{t}=\left(I_{n}-\tilde{\rho}^{\text {down }} A E B_{t}-\tilde{\rho}^{u p} \hat{A}^{\prime} T B_{t}\right) \cdot \Delta y_{t}-X_{t} \cdot \tilde{\beta} .
$$

The sum of the quasi-log-likelihood evaluated at the MLE, $\hat{\theta}_{I}$, for the inverted model is: $\mathscr{L}_{I}=\sum_{t=1}^{T} \ell_{t, I}\left(\hat{\theta}_{I}\right)$.
Following Wooldridge (2010), let's define the estimator for the variance of the KL divergence as:

$$
\hat{\eta}^{2}=\frac{1}{T} \cdot \sum_{t=1}^{T}\left(\ell_{t, B}\left(\hat{\theta}_{B}\right)-\ell_{t, I}\left(\hat{\theta}_{I}\right)\right)^{2}
$$

Then, the Vuong Model Selection Statistic, VMS, is:

$$
\begin{aligned}
V M S & =T^{-1 / 2} \cdot \frac{\left(\mathscr{L}_{B}-\mathscr{L}_{I}\right)}{\hat{\eta}} \\
& =\frac{\frac{1}{T} \cdot \sum_{t=1}^{T}\left(\ell_{t, B}\left(\hat{\theta}_{B}\right)-\ell_{t, I}\left(\hat{\theta}_{I}\right)\right)}{\sqrt{\frac{\frac{1}{T} \cdot \sum_{t=1}^{T}\left(\ell_{t, B}\left(\hat{\theta}_{B}\right)-\ell_{t, I}\left(\hat{\theta}_{I}\right)\right)^{2}}{T}}} \stackrel{d}{\rightarrow} N(0,1)
\end{aligned}
$$

where the standard normal distribution holds under:

$$
H_{0}: \mathbb{E}\left[\ell_{t, B}\left(\theta_{B}^{*}\right)\right]=\mathbb{E}\left[\ell_{t, I}\left(\theta_{I}^{*}\right)\right]
$$

where $\theta_{B}^{*}$ and $\theta_{I}^{*}$ are the pseudo-true values of the parameters. Basically, the null hypothesis is saying that the two potentially misspecified models fit the data equally well. Notice that the test is super easy to implement: 1) define the difference: $\hat{d}_{t}=\ell_{t, B}\left(\hat{\theta}_{B}\right)-\ell_{t, I}\left(\hat{\theta}_{I}\right) ; 2$. Regress $\hat{d}_{t}$ on unity; 3 . Run a t-test to verify that the average of the difference is statistically different from zero. We reject the null hypothesis in favor of a better fit to the data of the baseline model if $\hat{d}_{t}$ is statistically greater than zero. Notice that if this happens it
does not mean that the baseline model is correctly specified (although it could be), however, we can conclude that the baseline model fits better in terms of expected likelihood.
The value we obtain is VMS $=0.033$ which is clearly not statistically different from zero. Even if positive sign of the statistics points at a better fit of the baseline model against the inverted one, there is not enough statistical evidence to claim that the baseline outperforms on average the inverted model.

## D. 2 Output Effect of Fiscal Plans in the Inverted Model

We report here the estimated posterior distributions of the ATE, ADE and ANE for fiscal adjustment plans obtained from the estimates of Equation (7) (inverted model).

Table VIII: Average Total, Direct and Network Effects of Fiscal Consolidations in the United States

| Inverted Model-Equation (7) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbb{E}(\theta)$ | $\%$ | $\sqrt{\mathbb{V}(\theta)}$ | $\operatorname{Pr}(\theta<0)$ | $5 \%$ | $10 \%$ | $16 \%$ | $50 \%$ | $84 \%$ | $90 \%$ | $95 \%$ |  |  |  |  |  |  |
| $A T E_{T B}$ | -1.148 | 1 | 1.034 | 0.872 | -2.909 | -2.481 | -2.162 | -1.107 | -0.131 | 0.140 | 0.480 |  |  |  |  |  |  |
| $A D E_{T B}$ | -0.848 | 0.74 | 0.756 | 0.872 | -2.106 | -1.819 | -1.593 | -0.835 | -0.101 | 0.107 | 0.375 |  |  |  |  |  |  |
| $A N E_{T B}$ | -0.300 | 0.26 | 0.290 | 0.872 | -0.828 | -0.682 | -0.572 | -0.263 | -0.029 | 0.030 | 0.102 |  |  |  |  |  |  |
| $A T E_{E B}$ | 0.522 | 1 | 0.337 | 0.064 | -0.048 | 0.096 | 0.203 | 0.536 | 0.847 | 0.936 | 1.046 |  |  |  |  |  |  |
| $A D E_{E B}$ | 0.491 | 0.94 | 0.318 | 0.064 | -0.044 | 0.089 | 0.188 | 0.501 | 0.799 | 0.886 | 0.990 |  |  |  |  |  |  |
| $A N E_{E B}$ | 0.031 | 0.06 | 0.032 | 0.064 | -0.002 | 0.002 | 0.005 | 0.024 | 0.059 | 0.073 | 0.091 |  |  |  |  |  |  |

Table VIII: descriptive statistics of posterior distributions of Average Effects of a 2 years, $1 \%$ magnitude fiscal adjustment plan. 2 years means that results are calculated by cumulating the effect of the first year of the plan and then the second one. The style of the plan is simulated from a distribution which mimics the observed one; see Appendix C. 3 for technical details. Columns: $\mathbb{E}(\theta)$ is the expected value of the posterior distribution; \% is the share of ATE represented by $A D E$ and $A N E . \sqrt{\mathbb{V}(\theta)}$ is the standard deviations of the posterior distribution; $\operatorname{Pr}(\theta<0)$ is the probability of negative values, calculated by integrating the posterior distribution; " $p \%$ " is the $p$-th percentile of the posterior distribution.

The most important thing to notice is that the ANE of EB plans accounts for only $6 \%$ of their ATE, against the $12 \%$ of the baseline model. The relevance of ANE of TB plans is basically unaffected, diminishing only by $1 \%$ relative to the baseline (from $27 \%$ of the ATE to $26 \%$ ). The statistical significance of the ANE of TB plans declines, since the posterior distribution shrinks towards zero.

## E A Potential Theoretical Framework

We show here the theoretical framework which we have in mind when we refer to the theoretical transmission of demand and supply shocks. The model is a
slight modification of Acemoglu, Akcigit, and Kerr (2016), which we adapted to allow for the propagation of a production tax.

The model considers a perfectly competitive economy with $n$ sectors, where the market clearing condition for the generic industry $i$ is:

$$
\begin{equation*}
y_{i}=c_{i}+\sum_{j=1}^{n} x_{j i}+G_{i} \tag{8}
\end{equation*}
$$

where $c_{i}$ is household's consumption of good produced by industry $i ; x_{i j}{ }^{39}$ is the quantity of goods produced in industry $j$ used as inputs by industry $i$; $G_{i}$ are government purchases.

$$
\begin{equation*}
\sum_{i=1}^{n} p_{i} G_{i}=T+\tau \sum_{i=1}^{n} p_{i} y_{i} \tag{9}
\end{equation*}
$$

Each sector solves the following profit maximization problem:

$$
\max _{l_{i},\left\{x_{i j}\right\}_{j=1}^{n}}(1-\tau) \cdot p_{i} \cdot(\underbrace{l_{i}^{\alpha_{i}^{l}} \cdot\left(\prod_{j=1}^{n} x_{i j}^{\alpha_{i j}}\right)^{\rho}}_{y_{i}})-w l_{i}-\sum_{j=1}^{n} p_{j} x_{i j}
$$

where $\tau$ is a sales/production tax which mimics an excise tax. ${ }^{40}$ Notice that the production function is similar to the one in Acemoglu, Carvalho, et al. (2012) and Carvalho (2014). All alpha's are non negative, and we assume constant return to scale: $\alpha_{i}^{l}+\rho \cdot \sum_{j=1}^{n} a_{i j}=1$. Notice here, that thanks to the Cobb-Douglas specification, $\rho$ can be interpreted as the share of intermediates in production.

The economy is populated by a representative agent, who maximizes utility subject to a budget constraint:

$$
\max _{l,\left\{c_{i}\right\}_{i=1}^{n}}(1-l)^{\lambda} \cdot \prod_{i=1}^{n} c_{i}^{\beta_{i}} \quad \text { s.t. } \sum_{i=1}^{n} p_{i} c_{i} \leq w l
$$

[^26]with $\sum_{i=1}^{n} \beta_{i}=1$.
Firms and households take all prices as given, and the market-clearing conditions are satisfied in the goods market and the labor market. Government actions are taken as given and the wage is chosen as a numeraire $(w=1)$.

We do not explicitly model a government budget constraints, since during years of fiscal consolidations, spending cuts are not compensated by tax reductions and viceversa. For simplicity we also do not model government debt and deficit.

Households. The household problems returns the following equilibrium conditions:

$$
\begin{aligned}
\frac{p_{i} \cdot c_{i}}{\beta_{i}} & =\frac{p_{j} \cdot c_{j}}{\beta_{j}} \quad \forall i, j \\
l & =\frac{1}{1+\lambda} \\
c_{i} & =\frac{\beta_{i}}{p_{i}} \cdot \frac{1}{1+\lambda} \quad \forall i \\
\sum_{i=1}^{n} p_{i} \cdot c_{i} & =\frac{1}{1+\lambda}
\end{aligned}
$$

Therefore, in equilibrium we have:

$$
d \log c_{i}=-d \log p_{i} \quad \forall i
$$

that is, percent changes in consumption of good $i$ only depend on percent changes in the price of the same good (with Cobb-Douglas utility income and substitution effects cancel out).

Firms Firms maximize profits and in equilibrium the following FOCs hold true:

$$
\begin{aligned}
(1-\tau) \cdot p_{i} \cdot \rho \cdot a_{i j} \cdot \frac{y_{i}}{x_{i j}} & =p_{j} \\
(1-\tau) \cdot p_{i} \cdot \rho \cdot a_{i}^{l} \cdot \frac{y_{i}}{l_{i}} & =1 \\
y_{i} & =l_{i}^{\alpha_{i}^{l}} \cdot\left(\prod_{j=1}^{n} x_{i j}^{\alpha_{i j}}\right)^{\rho}
\end{aligned}
$$

Acemoglu, Akcigit, and Kerr (2016) notes that solving the dual problem (cost minimization) and obtaining the unit cost function is beneficial to the analysis.

The unit cost function is equal to:

$$
C\left(p_{1}, \ldots, p_{n}\right)=\underbrace{\left(\frac{1}{a_{i}^{l}}\right)^{a_{i}^{l}} \cdot\left(\prod_{j=1}^{n}\left(\frac{1}{\rho \cdot a_{i j}}\right)^{a_{i j}}\right)^{\rho}}_{:=B_{i}}\left(\prod_{j=1}^{n} p_{j}^{a_{i j}}\right)^{\rho}
$$

Because of perfect competition, price equals marginal cost. Therefore:

$$
(1-\tau) \cdot p_{i}=C\left(p_{1}, \ldots, p_{n}\right)
$$

By $\log$ differentiating the above expression, we have:

$$
d \log p_{i}=\rho \cdot \sum_{j=1}^{n} a_{i j} \cdot d \log p_{j}+\frac{\tau}{1-\tau} d \log \tau
$$

The above expression implies that prices are affected only by changes in the production $\operatorname{tax} \tau$. Moreover, from profit maximiation we also have:

$$
\rho \cdot a_{i j}=\frac{1}{1-\tau} \cdot \frac{p_{j} \cdot x_{i j}}{p_{i} \cdot y_{i}} \propto \frac{\operatorname{SALES}_{j \rightarrow i}}{\operatorname{SALES}_{i}} .
$$

In other word, if sector $i$ is affected by a tax shock, the effect is propagated downstream to the customers, via $x_{i j}$. This should be clear if we substitute the firm's FOC condition into the previous expression:

$$
d \log p_{i}=\frac{1}{1-\tau} \cdot \sum_{j=1}^{n} \frac{p_{j} \cdot x_{i j}}{p_{i} \cdot y_{i}} \cdot d \log p_{j}+\frac{\tau}{1-\tau} d \log \tau
$$

## E. 1 Network effect of a tax shock

We want to know what is the output effect of a change in the production tax. In order to do so, we need to look at the resource constraint (assuming for
simplicity that $G_{i}=0$ for all sectors):

$$
\begin{aligned}
& y_{i}=c_{i}+\sum_{j=1}^{n} x_{j i} \\
& \frac{y_{i}}{c_{i}}=1+\sum_{j=1}^{n} \frac{x_{j i}}{c_{i}} \text { plug in: } x_{j i}=(1-\tau) \cdot p_{j} \cdot \rho \cdot a_{j i} \cdot \frac{y_{j}}{p_{i}} \text { (Firm FOC) } \\
& \frac{y_{i}}{c_{i}}=1+(1-\tau) \cdot \rho \sum_{j=1}^{n} a_{j i} \cdot \frac{p_{j} \cdot y_{j}}{p_{i} \cdot c_{i}} \text { plug in: } c_{i}=\frac{\beta_{i}}{\beta_{j}} \cdot \frac{p_{j} \cdot c_{j}}{p_{i}} \text { (HH FOC) } \\
& \frac{y_{i}}{c_{i}}=1+(1-\tau) \cdot \rho \sum_{j=1}^{n} a_{j i} \cdot \frac{\beta_{i}}{\beta_{j}} \cdot \frac{y_{j}}{c_{j}} \text { Denote by: } \theta_{i}:=y_{i} / c_{i} \\
& \theta_{i}=1+(1-\tau) \cdot \rho \sum_{j=1}^{n} \underbrace{a_{j i} \cdot \frac{\beta_{i}}{\beta_{j}}}_{m_{i j}} \cdot \theta_{j}
\end{aligned}
$$

Denote by $M:=\left[m_{i j}\right]_{i, j=1, \ldots, n}$. Then, in matrix notation the above expression becomes:

$$
\boldsymbol{\theta}=\mathbf{1}_{n}+(1-\tau) \cdot \rho \cdot M \cdot \boldsymbol{\theta} \Longrightarrow \boldsymbol{\theta}=\left(I_{n}-(1-\tau) \cdot \rho \cdot M\right)^{-1} \cdot \mathbf{1}_{n}
$$

Notice that the equilibrium level of the output-to-consumption ratio, $\theta_{i}$, has a nice analytical form which, however, depends on $\tau$. Therefore, when $\tau$ changes, also this ratio changes and we don't have $d \log y_{i}=d \log c_{i}$ as in Acemoglu, Akcigit, and Kerr (2016).

Differentiating the above expression yields:

$$
\begin{aligned}
d \boldsymbol{\theta} & =\frac{\partial\left(I_{n}-(1-\tau) \cdot \rho \cdot M\right)^{-1}}{\partial \tau} \cdot \mathbf{1}_{n} d \tau \\
& =-\rho \cdot\left(I_{n}-(1-\tau) \cdot \rho \cdot M\right)^{-1} \cdot M \cdot\left(I_{n}-(1-\tau) \cdot \rho \cdot M\right)^{-1} \cdot \mathbf{1}_{n} d \tau
\end{aligned}
$$

Using the $d \log$ notation:
$d \log \boldsymbol{\theta}=-\underbrace{\tau \cdot \rho \cdot \Theta^{-1} \cdot\left(I_{n}-(1-\tau) \cdot \rho \cdot M\right)^{-1} \cdot M \cdot\left(I_{n}-(1-\tau) \cdot \rho \cdot M\right)^{-1}}_{:=F} \cdot \mathbf{1}_{n} d \log \tau$
where $\Theta=\operatorname{diag}\left(\theta_{1}, \ldots, \theta_{n}\right)$. Recalling the definition of $\theta_{i}$, we have:

$$
d \log \boldsymbol{y}=d \log \boldsymbol{c}-F \cdot \mathbf{1}_{n} \cdot d \log \tau \Longrightarrow d \log y_{i}=d \log c_{i}-\phi_{i} \cdot d \log \tau
$$

where $\phi_{i}$ is the i-th element of vector $F \cdot \mathbf{1}_{n}$. Notice that if $\tau$ were fixed (i.e. $d \log \tau=0$ ), percent changes in consumption would be equal to the one of output, as in Acemoglu, Akcigit, and Kerr (2016).

At this point we can find the relationship between output changes and tax shocks. Consider the following three equations we derived earlier:

$$
\left\{\begin{aligned}
d \log y_{i} & =d \log c_{i}-\phi_{i} \cdot d \log \tau \\
d \log c_{i} & =-d \log p_{i} \\
d \log p_{i} & =\rho \cdot \sum_{j=1}^{n} a_{i j} \cdot d \log p_{j}+\frac{\tau}{1-\tau} d \log \tau
\end{aligned}\right.
$$

Combining the three equations above yields the following expression:

$$
\begin{aligned}
d \log y_{i} & =\rho \cdot \sum_{j=1}^{n} a_{i j} \cdot d \log y_{j}-\underbrace{\left(\phi_{i}+\frac{\tau}{1-\tau}-\rho \cdot \sum_{j=1}^{n} \phi_{j} \cdot a_{i j}\right)}_{=\psi_{i}>0} \cdot d \log \tau \\
& =\rho \cdot \sum_{j=1}^{n} a_{i j} \cdot d \log y_{j}-\psi_{i} \cdot d \log \tau
\end{aligned}
$$

which is Equation (4) in the paper.

## E. 2 Network effect of a spending shock

Suppose now that $\tau=0$ and that the government reduces its purchases from all sectors (i.e. $d \log G_{i}<0$ ). We want to find the relationship between the percent change in output, $d \log y_{i}$ and percent changes in government purchases $d \log G_{i}$.

Consider the resource constraint of the economy:

$$
\begin{aligned}
y_{i} & =c_{i}+G_{i}+\sum_{j=1}^{n} x_{j i} \quad \text { Log-differentiate } \\
d \log y_{i} & =\frac{c_{i}}{y_{i}} \underbrace{d \log c_{i}}_{=0}+\frac{G_{i}}{y_{i}} d \log G_{i}+\sum_{j=1}^{n} \frac{x_{j i}}{y_{i}} \cdot d \log x_{j i} \quad(\text { Firm FOC }) x_{j i}=p_{j} \rho a_{j i} \frac{y_{j}}{p_{i}} \\
d \log y_{i} & =\frac{G_{i}}{y_{i}} d \log G_{i}+\rho \cdot \sum_{j=1}^{n} \underbrace{a_{j i} \cdot \frac{p_{j} y_{j}}{p_{i} y_{i}}}_{:=\hat{a}_{j i}} d \log x_{j i} \\
d \log y_{i} & =\frac{G_{i}}{y_{i}} d \log G_{i}+\rho \cdot \sum_{j=1}^{n} \hat{a}_{j i} \cdot d \log x_{j i}
\end{aligned}
$$

From the firm's FOC, we have:

$$
d \log y_{i}=\underbrace{d \log p_{j}}_{0}+d \log x_{j i}-\underbrace{d \log p_{i}}_{=0}
$$

therefore we can retrieve Equation (2):

$$
d \log y_{i}=\rho \cdot \sum_{j=1}^{n} \hat{a}_{j i} \cdot d \log y_{j}+\frac{G_{i}}{y_{i}} d \log G_{i} .
$$


[^0]:    *This paper was presented during IAAE 2018 conference meeting, HSE and Bocconi seminars. We are grateful to participants for useful comments; in particular, we thank Roberto Perotti, Hashem Pesaran, Lung-Fei Lee and Valerie Ramey for useful comments.
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[^1]:    ${ }^{1}$ Steigum and Thøgersen (2003) show in a two-sector model with overlapping generations that fiscal deficits benefit only present generations. Sooner or later the fiscal austerity is needed.
    ${ }^{2}$ On April 25th 2020, in an article entitled "After the disease, the debt", The Economist wrote: "... governments should prepare for the grim business of balancing budgets later in the decade."
    ${ }^{3}$ For a literature survey on sovereign debt, see Panizza, Sturzenegger, and Zettelmeyer (2009) and Reinhart and Rogoff (2009).

[^2]:    ${ }^{4}$ Key suppliers in the network are: Fabricated Metal Products, Primary Metals, Wholesale Trade, Plastic and Rubber Products, Chemical Products, Real Estate, Administrative Services, Miscellaneous Professional, Scientific and Technical Services.

[^3]:    ${ }^{5}$ Other recent theoretical and empirical contributions include, but not limited to Baqaee and Farhi (2018), Baqaee and Farhi (2019a), Baqaee and Farhi (2019b), Barrot and Sauvagnat (2016), Boehm, Flaaen, and Pandalai-Nayar (2019). Carvalho and Tahbaz-Salehi (2019) summarize the literature providing both theoretical foundation for production networks as a propagation channel as well as evidence from growing empirical literature.

[^4]:    ${ }^{6}$ In the above expression $j \geq 1$ since any policy revision introduced upon implementation $(j=0)$ is no longer a part of an anticipated shock; in fact, it is a new unanticipated component.

[^5]:    ${ }^{7}$ Concerning expenditure shocks, we emphasize that Alesina, Barbiero, et al., 2017 disentangle transfers from taxes and government spending. They show that the difference in output responses is not driven by the inclusion of transfers among other public spending measures.

[^6]:    ${ }^{8}$ We thank Valerie Ramey for bringing up this point.
    ${ }^{9}$ Unlike us Zubairy (2014) uses a dynamic stochastic general equilibrium framework to investigate the transmission mechanism of fiscal multipliers. Moreover, Evans, Honkapohja, and Mitra (2022) show that spending multiplier depends both on the current state of expectations, as well as the size and duration of the expenditure increase.
    ${ }^{10}$ See Appendix A for details on the data and the regression equation.

[^7]:    ${ }^{11}$ We measure procurement spending as done in Briganti and Sellemi (2022).

[^8]:    ${ }^{12}$ EB plans also increase payroll taxes, however, they account for a minor part of the total EB plan (see again Figure 2).

[^9]:    ${ }^{13}$ The modifications come from the inclusion of a production tax and an extra parameter in the production function. We remand to the Appendix E for the detailed derivations.
    ${ }^{14}$ Because of constant return to scale we have $\alpha_{i}^{l}+\rho \cdot \sum_{j=1}^{n} a_{i, j}=1$ for all sectors.

[^10]:    ${ }^{15}$ See Appendix E.

[^11]:    ${ }^{16}$ See Appendix E for derivation.

[^12]:    ${ }^{17}$ We denote the parameters that we estimate in blue.

[^13]:    ${ }^{18}$ Bayesian MCMC is also more appealing than MLE for some quite technical reasons. However, we save these details for Appendix C.3. An alternative approach can be a generalized moments estimator offered by Kelejian and Prucha (1999).

[^14]:    ${ }^{19}$ We use average output shares in years of TB fiscal consolidation for aggregating TB effects. We use average output shares in years of EB fiscal consolidation for aggregating EB effects

[^15]:    ${ }^{20}$ In doing so we draw all the parameters jointly from each step of the Markov Chain to take into account the potential correlation among the parameters' distributions.
    ${ }^{21}$ See Appendix, section C.4, for further information on the empirical distribution of the style of US fiscal plans.

[^16]:    ${ }^{22}$ From Table III, we have: $|-0.380-0.043| /|-1.397-0.370| \approx 25 \%$ in the baseline model and $|-0.300-0.031| /|-1.148-0.522| \approx 20 \%$ in the inverted model.
    ${ }^{23}$ This is in line with Alesina, Favero, and Giavazzi (2020).

[^17]:    ${ }^{24}$ For more on in-degrees and out-degrees of the industrial network see Acemoglu, Carvalho, et al. (2012) and Carvalho and Tahbaz-Salehi (2019).

[^18]:    ${ }^{25}$ Derivation and details of the Vuong test are outlined in Appendix D.1.
    ${ }^{26}$ Tables of results are reported in Appendix D.2.

[^19]:    ${ }^{27}$ The choice of 0.0001 is motivated by the presence of several values of $A$ which are close to zero but not exactly zero. The choice of 0.03 is motivated by the presence of only a few values above this threshold. In general, tweaking these numbers still allows observing such a visual pattern of matrix $A$.

[^20]:    ${ }^{28}$ Actually, slightly more dots are located more South-West than the original simulation; this is not surprising if we think that we are moving the large elements of the main diagonal (see heat-map 9) outside of it, thus mechanically inflating the indirect spillover of the sector receiving the main diagonal entry.

[^21]:    ${ }^{29}$ Their decision is justified by the fact that value-added is adjusted for energy costs, nonmanufacturing input, and inventory changes which are all outside of the general equilibrium model which provides the theoretical underpinning to their empirical strategy.
    ${ }^{30}$ Our definition of Government encompasses both Federal and State\&Local government spending. We therefore exclude here Government Enterprises, which instead are considered as part of the industrial network.
    ${ }^{31}$ We thank Roberto Perotti for this point.
    ${ }^{32}$ We use the Make and Use tables of year 1997, which is the closest to the occurrence of fiscal plans. Nevertheless, notice that I-O matrices are fairly stable over time.

[^22]:    ${ }^{33}$ Notice that a big assumption is made in the construction of this matrix: if industry $i$ has adjusted market share of production of commodity $K, O U T_{i \rightarrow K} /\left(C_{K} \cdot \theta_{K}\right)$ equal to, say $10 \%$, then it is assumed that if industry $j$ purchases $z:=\mathrm{INP}_{K \rightarrow j}$ dollars of commodity $K$, then $10 \%$ of $z \$$ come from industry $i$. This must be true on average but it might not be exactly true case by case.

[^23]:    ${ }^{34}$ We are grateful to Hashem Pesaran for making us aware of this.
    ${ }^{35}$ We thank Lung-Fei Lee for pointing this out.

[^24]:    ${ }^{36}$ Results for the inverted model, Equation (7)) are symmetric to the baseline case.
    ${ }^{37} k$ in our baseline is $n$ fixed effects plust 6 fiscal adjustment components (unexpected, announced and future for both TB and EB plans) plus 2 year dummies for 2008 and 2009.

[^25]:    ${ }^{38}$ See Section 13.11.2 - Model Selection Tests.

[^26]:    ${ }^{39}$ In Equation (8) we actually have $x_{j i}$, that is, the amount of good $i$ used as input by industry $j$; we then sum over the $j$-s to obtain the total demand of good $i$ from all the industries.
    ${ }^{40}$ For example, an excise is a special type of sales tax, which is sector-specific. Excise tax might be of two types: ad valorem (percentage of values of a good) and specific (tax paid per unit). The excise tax may be paid by the producer, retailer, and consumer. Moreover, it might be taken on federal, state, and local levels.

