

The Network Effects of Fiscal Adjustments

Edoardo Briganti* Carlo A. Favero[†]
Madina Karamysheva[‡]

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Abstract

We investigate the effects of fiscal consolidations in the United States and their propagation through the production network. Using a narrative approach, we identify exogenous fiscal adjustments and employ a spatial autoregression (SAR) model to separate the total effects of these adjustments into direct and network components. Our analysis reveals that tax-based consolidations have a more pronounced recessionary impact than expenditure-based ones, with approximately 27% of the total effect of tax-based consolidations attributable to network spillovers, compared to 11% for expenditure-based plans. A quarter of this difference in their total output effect is attributable to the stronger network propagation of tax increases compared to government spending cuts.

Keywords: industrial networks, fiscal adjustment plans, output growth, applied spatial econometrics.

JEL codes : E60, E62.

*University of California San Diego, ebrigant@ucsd.edu

[†]Deutsche Bank Chair, IGIER-Bocconi, and CEPR (full address: IGIER and Dept. of Finance, Universit Commerciale Luigi Bocconi, via Roentgen 1, 20136 Milano Italy), tel: +390258363306; carlo.favero@unibocconi.it

[‡]NRU Higher School of Economics, (full address: Pokrovsky Boulevard 11, Moscow, Russian Federation); mkaramysheva@hse.ru

1 Introduction

The interconnectedness of countries, financial systems, and industries is crucial for understanding the transmission of policy shocks. This paper investigates the propagation mechanisms of fiscal consolidations through the industrial production network with a particular focus on the differential impacts of tax-based (TB) and expenditure-based (EB) adjustments. We explore how fiscal shocks when filtered through the production network, lead to varied macroeconomic outcomes.

Economic principles and best practices advocate for deficit spending during economic downturns and surplus budgeting in times of prosperity, aiming for overall fiscal balance. In accordance with this principle, the fiscal stimuli implemented in response to the COVID-19 pandemic have led to a deterioration in public finances, increasing the likelihood of austerity measures in several countries.¹ In this context, gaining a comprehensive understanding of the mechanisms through which fiscal consolidations are transmitted is essential for their efficient implementation.

Our analysis is situated within the broader literature on fiscal policy and its effects on economic output. It is well-documented that consolidations achieved through spending cuts generally have a less detrimental impact on economic growth than those achieved through increased taxation (Ramey (2019)). This distinction is crucial for policymakers charged with designing and implementing fiscal strategies that minimize loss to economic output while ensuring fiscal stability.

Despite the importance of this finding, the literature has devoted little attention to the reasons behind this asymmetrical effect on the economy. In this paper, we propose production networks as a potential explanation for the asymmetric output responses to tax-based (TB) versus expenditure-based (EB) fiscal consolidations. In particular, we investigate the effects of fiscal consolidations in the United States from 1978 to 2014 at an industry level, covering 62 interconnected industries. Our findings indicate that TB fiscal adjustments result in a significant recessionary output multiplier over two years (about 1.4%), while the effects of EB fiscal plans are not statistically different from zero. These results align with existing literature (see Guajardo, Leigh, and Pescatori (2014), Alesina, Favero, and Giavazzi (2015)) but extend the

¹On April 25, 2020, in an article entitled “After the disease, the debt”, The Economist wrote: “... governments should prepare for the grim business of balancing budgets later in the decade.” Furthermore, Steigum and Thøgersen (2003) demonstrate in a two-sector overlapping generations model that fiscal deficits primarily benefit present generations, making fiscal austerity a necessary measure sooner or later.

analysis by incorporating the network dimension.

We employ a spatial autoregressive (SAR) model (LeSage and Pace, 2009), with spatial distance defined by the input-output production network.² Specifically, we utilize a panel version of the SAR model, which not only considers the spatial-industry dimension but also the time dimension. To accommodate the varying volatility in growth rates across sectors, we allow for heteroskedasticity by employing both the maximum likelihood method, as suggested by Aquaro, Bailey, and Pesaran (2021), and Bayesian MCMC, as described in LeSage and Pace (2009).

We decompose the aggregate effects of fiscal consolidations into direct and network effects. The direct effect measures the impact of the fiscal shock on each industry, while the network effect considers the spillover effects from other industries within the same production network. Our findings show that approximately 27% of the total effect of TB fiscal consolidations is due to network spillovers, in contrast to a more modest 11% for EB plans. This variance in network effects contributes to about one-fourth of the difference in the total effects of TB and EB plans, underscoring the importance of production networks in explaining the asymmetric output response to fiscal consolidations.

Furthermore, our analysis highlights the role of key suppliers in the economy in propagating tax shocks through the network.³ This finding is consistent with research on the upstream propagation of monetary policy shocks by Ozdagli and Weber (2017) and Di Giovanni and Hale (2022), emphasizing the significance of incorporating production networks into fiscal policy analysis.

In summary, this paper enhances the understanding of how fiscal adjustment plans are transmitted by spotlighting the role of production networks in amplifying and disseminating fiscal shocks. Through an examination of the direct and network effects of fiscal consolidations, we offer insights into the mechanisms that contribute to the asymmetric output response to TB and EB plans.

Related Literature

First, our paper relates to the literature on fiscal consolidations, including works by Perotti (1999), Guajardo, Leigh, and Pescatori (2014), Alesina,

²Given that input-output (IO) coefficients typically remain stable over time, we use a static IO matrix. This approach assumes that market participants respond based on the existing intensity of production network connections. As mentioned in Di Giovanni and Hale (2022), this assumption is particularly suitable at the industry level.

³Key suppliers in the network include: Fabricated Metal Products, Primary Metals, Wholesale Trade, Plastic and Rubber Products, Chemical Products, Real Estate, Administrative Services, and Miscellaneous Professional, Scientific, and Technical Services.

Favero, and Giavazzi (2015), Pappa, Sajedi, and Vella (2015), Alesina, Barbiero, et al. (2017), Bandeira et al. (2018), Born, Müller, and Pfeifer (2020), Beetsma et al. (2021), Carrière-Swallow, David, and Leigh (2021), Chang et al. (2021), Gabriel, Klein, and Pessoa (2023). Unlike these studies, which primarily focus on country-level analyses, we investigate the network effects of fiscal consolidations within a panel of U.S. industries.

Alesina, Barbiero, et al. (2017) provide a theoretical framework for understanding the stronger impact of tax-based (TB) fiscal consolidations. Other studies, such as Brinca et al. (2021) and Karamysheva (2022), explore various channels through which fiscal consolidations affect the economy. Our work adds to this literature by offering a network-based perspective on the asymmetric effects of TB and EB fiscal consolidations.

Second, our research is related to the literature on production networks, exemplified by the seminal works of Gabaix (2011) and Acemoglu, Carvalho, et al. (2012). These studies highlight the role of networks in amplifying economic shocks.⁴ Recent contributions in this area have explored various aspects of network dynamics, including the asymmetric propagation of shocks (Acemoglu, Akcigit, and Kerr, 2016, the impact of fiscal spending multipliers (Flynn, Patterson, and Sturm (2022), Barattieri, Cacciatore, and Traum (2023), Bouakez, Rachedi, and Santoro (2023), Devereux, Gente, and C. Yu (2023)). Our paper extends this analysis by adopting a spatial framework to examine the network transmission of fiscal adjustment plans, drawing on approaches used by Ozdagli and Weber (2017) and Di Giovanni and Hale (2022) in the context of monetary policy.⁵

Third, we contribute to the literature on the industry-level effects of fiscal policy. Studies by Ramey and Shapiro (1999), Perotti (2007), and Nekarda and Ramey (2011) have examined the impact of government spending on specific sectors. Cox et al. (2020) explore public procurement contracts and uncover a significant sectoral bias in government spending. Auerbach, Gorodnichenko, and Murphy (2020) analyze city-level data on local defense public procure-

⁴Other recent theoretical and empirical contributions include, but are not limited to, Baqaee and Farhi (2018), Baqaee and Farhi (2019a), Baqaee and Farhi (2019b), Barrot and Sauvagnat (2016), Boehm, Flaaen, and Pandalai-Nayar (2019), and Carvalho and Tahbaz-Salehi (2019). Carvalho and Tahbaz-Salehi (2019) summarizes the literature, providing both a theoretical foundation for production networks as a propagation channel as well as evidence from the growing empirical literature.

⁵A separate stream of literature focuses on the pure geographical spillover effects of fiscal policy, including Corsetti, Meier, and Müller (2010), Faccini, Mumtaz, and Surico (2016), Alloza, Burriel, and Pérez (2019), Klein and Linnemann (2021), Cacciatore and Traum (2022), and Ilori, Paez-Farrell, and Thoenissen (2022). In contrast, our focus is on industry spillovers.

ment, discovering substantial fiscal spillovers among industries. These findings underscore the importance of considering sectoral heterogeneity in fiscal policy effects. We build on this work by analyzing the transmission of fiscal adjustment plans across a comprehensive set of industries within a production network framework.

In summary, our paper provides new insights into the propagation mechanisms of fiscal adjustment plans, highlighting the significance of production networks in determining their effects.

The paper is organized as follows: Section 2 outlines how fiscal adjustment plans indicate exogenous fiscal consolidation policies and examines their aggregate effects, offering a theoretical rationale for the transmission mechanism. Section 3 details our findings. Section 4 addresses robustness checks, and Section 5 concludes.

2 Fiscal Adjustments Plans in the US

Measuring the propagation of fiscal adjustments requires the identification of an exogenous demand and supply shocks. Our identification strategy thus relies on the narrative analysis of fiscal adjustment plans. This strategy is a recent innovation in the fiscal policy literature and employs narrative exogenous shocks as a proxy for fiscal consolidation policies. This strategy was introduced in Alesina, Favero, and Giavazzi (2015) to take into account the fact that fiscal adjustments are implemented through multi-year plans with both an intertemporal and an intratemporal dimension.

The intratemporal dimension refers to the fact that fiscal consolidations are implemented with a mix of tax increases and spending cuts. Tax and the expenditure components of the adjustments are correlated since governments decide first on the size of the adjustment, and then on its composition in terms of expenditures and revenues. The intertemporal dimension is important since fiscal adjustments are implemented via multi-year plans with measures upon announcement (the unanticipated component of the plan) and measures announced for subsequent years (the anticipated component of the plan). In particular, each country has a specific “recipe” to implement fiscal consolidations: some countries prefer to unexpectedly raise taxes without cutting expenditures, while others announce large future cuts in spending and only marginally increase taxes. Alesina, Favero, and Giavazzi (2015) refer to this as the country-specific “style of the plan”.

These complications make identifying pure and isolated tax hikes and spending cuts during years of fiscal consolidation a difficult, if not impossible, task.

Fiscal plans provide an effective tool to circumvent these difficulties when studying austerity policies.

2.1 Modeling Fiscal Plans:

From a mathematical standpoint, plans are sequences of fiscal corrections, announced at time t and implemented between t and $t + K$, where K is the anticipation horizon. In each year t , two types of fiscal corrections are possible:

1. The unanticipated fiscal shock, that is, the surprise change in the primary surplus at time t , which we denote by:

$$f_t^u := tax_t^u + exp_t^u,$$

where tax_t^u is the surprise increase in taxes announced and implemented at time t , while exp_t^u is the surprise reduction in government expenditure also announced and implemented at time t .

2. The anticipated fiscal shock: the change in the primary surplus at time t , which had already been announced in the previous years and is either implemented in year t or scheduled to happen within K years. In particular, we denote as $tax_{t,j}^a$ and $exp_{t,j}^a$ the tax and expenditure changes announced by the fiscal authorities at date t with an anticipation horizon of j years (*i.e.*, to be implemented in year $t + j$). Therefore, we further distinguish between:

- (a) The anticipated implemented shock: scheduled in the past and implemented in year t :

$$f_t^a := tax_{t,0}^a + exp_{t,0}^a$$

- (b) The anticipated future shocks: sum of scheduled tax and government spending changes which have to be implemented within K years from their announcement:

$$f_t^f := \sum_{j=1}^K tax_{t,j}^a + \sum_{j=1}^K exp_{t,j}^a.$$

In a fiscal adjustment database, as long as no policy revision takes place, the anticipated shocks roll over year-by-year. In formulae:

$$tax_{t,j}^a = \underbrace{tax_{t-1,j+1}^a}_{\text{Old shock, rolled over}} \quad exp_{t,j}^a = \underbrace{exp_{t-1,j+1}^a}_{\text{Old shock, rolled over}}.$$

However, if from one year to another, a policy revision takes place, then, the new anticipated future shock will embed such change:⁶

$$\begin{aligned} tax_{t,j}^a &= \underbrace{tax_{t-1,j+1}^a}_{\text{Old shock, rolled over}} + \underbrace{(tax_{t,j}^a - tax_{t-1,j+1}^a)}_{\text{Policy Revision}}, \quad \text{with } j \geq 1 \\ exp_{t,j}^a &= \underbrace{exp_{t-1,j+1}^a}_{\text{Old shock, rolled over}} + \underbrace{(exp_{t,j}^a - exp_{t-1,j+1}^a)}_{\text{Policy Revision}}, \quad \text{with } j \geq 1 \end{aligned}$$

We adopt the annual database on fiscal adjustment plans constructed by Alesina, Favero, and Giavazzi (2015) and consider only fiscal consolidations that occurred in the US from 1978 to 2014. They identify fiscal adjustments exogenous with respect to output fluctuations using a narrative identification method. This approach is similar to C. D. Romer and D. H. Romer (2010), who identify exogenous tax shocks from presidential speeches, congressional debates, budget documents, and congressional reports. From these documents, they identify the size, timing, and principal motivation for all major postwar tax policy actions. Legislated changes are then classified into two categories: 1) endogenous, if induced by short-run counter-cyclical concerns; 2) exogenous, if taken in response to the state of government debt (deficit-driven).⁷ As mentioned earlier, fiscal adjustment plans allow us to control for the intertemporal and intratemporal correlation, which we report in Table I:

⁶In the above expression $j \geq 1$ since any policy revision introduced upon implementation ($j = 0$) is no longer a part of an anticipated shock; in fact, it is a new unanticipated component.

⁷Concerning expenditure shocks, we emphasize that Alesina, Barbiero, et al., 2017 disentangle transfers from taxes and government spending. They show that the difference in output responses is not driven by the inclusion of transfers among other public spending measures.

Table I: Inter- and Intra-temporal correlation matrix of Fiscal Adjustments Plans in the US

	tax_t^u	$tax_{t,0}^a$	tax_t^f	exp_t^u	$exp_{t,0}^a$	exp_t^f
tax_t^u	1	0.041	0.570	0.596	-0.126	0.105
$tax_{t,0}^a$		1	0.038	0.098	0.361	0.310
tax_t^f			1	-0.047	0.019	0.180
exp_t^u				1	-0.050	0.014
$exp_{t,0}^a$					1	0.782
exp_t^f						1

Table I: linear correlation matrix of legislated changes in taxes and expenditure identified by the narrative analysis. Sample: annual data from 1978 to 2014 of US fiscal adjustment plans from Alesina, Favero, and Giavazzi (2015). In blue is reported the intra-temporal correlation (between each component of taxes and expenditures). In green is the inter-temporal correlation (within tax or expenditure component, but between components with different timing). In black we have a mix of the two: correlation between tax and expenditure components with different timing.

Notice from Table I that the intra-temporal correlation between unanticipated tax and unanticipated expenditure adjustments is 60% (blue figures in Table I). Similarly, the (inter-temporal) correlation between future and anticipated components of expenditure is 78% (green figures in Table I). As both the inter-temporal and the intra-temporal dimension matter, it is worth considering multi-year fiscal plans instead of individual measures of tax and government spending shocks.

Since this source of correlation confounds the effects of taxes and expenditures, we need to classify plans into mutually exclusive categories which can be simulated independently. We can then take into account the inter-temporal correlation within each category. To this end, we exploit the fact that not all the plans are the same. Some fiscal plans are designed to increase taxes more than cut expenditures and are labeled as TB (tax-based). On the contrary, those plans which rely more on expenditure cuts rather than tax hikes are labeled as EB (expenditure-based).

For instance, the criterion which determines whether a fiscal consolidation is labeled as TB can be written:

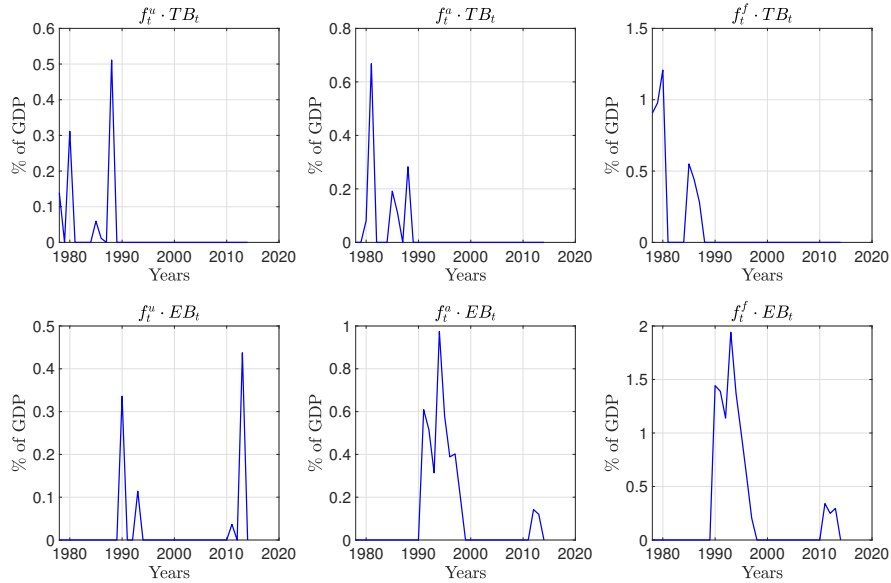
$$\underbrace{\left(tax_t^u + tax_{t,0}^a + \sum_{j=1}^K tax_{t,j}^a \right)}_{\text{overall tax hike in } t} > \underbrace{\left(exp_t^u + exp_{t,0}^a + \sum_{j=1}^K exp_{t,j}^a \right)}_{\text{overall expenditure cut in } t}. \quad (1)$$

Criterion (1) is saying that if the overall tax hike in year t exceeds the overall

spending cut, then we label year t as a year of TB fiscal consolidation. We keep track of these years by constructing two dummy variables, TB_t and EB_t , which are equal to one if year t is labeled as TB or EB, respectively. By construction, TB and EB plans are mutually exclusive. That is, EB and TB plans cannot occur simultaneously. This lets us simulate separately the effect of TB and EB plans while preserving, within each type of plan, the observed intra-temporal correlation between adjustments on government's revenues and expenditure.

Figure 1 plots our fiscal adjustment plans database. This contains all of the nominal changes in taxes and expenditure, scaled by GDP of the year before the consolidation occurs to avoid potential endogeneity issues. Moreover, the future component of the fiscal adjustment plan has a maximum anticipation horizon of three years (K). This is in line with the small numbers of occurrences of policy shifts anticipated four and five years ahead, and is consistent with the database in Pescatori et al. (2011). The top row of Figure 1 illustrates the three components of fiscal adjustments interacted with the dummy TB_t to identify the components of tax-based fiscal consolidations. The bottom row does the same for expenditure-based fiscal consolidations.

Figure 1: Fiscal Adjustments Plans - United States 1978-2014



We assess the goodness of our orthogonalization criterion (1), by showing in Figure 2 the share of tax increases and spending cuts of each total fiscal

adjustment, $f_t^u + f_t^a + f_t^f$.

Figure 2: Fiscal Adjustment Composition

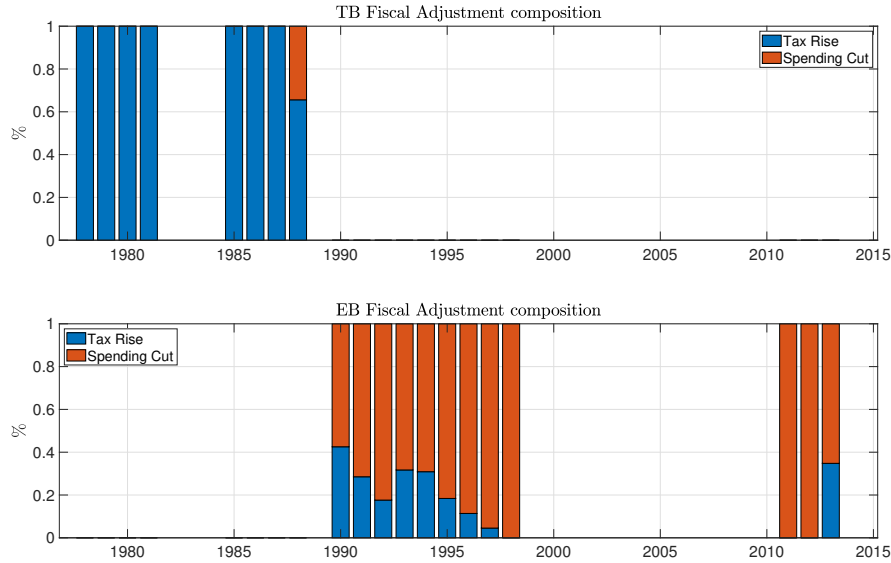


Figure 2 shows that the labeling of fiscal adjustment into EB or TB plans, by means of criterion (1), is never marginal: i. TB plans are all pure tax hikes except for the year 1988, which is the result of a hybrid fiscal plan with only 30% in spending cuts; ii. EB plans are mainly made up of spending cuts with only 20% of policy changes coming from a tax increase, on average. Figure 2 also illustrates the timing of fiscal consolidations in the US: i. there are two periods of TB fiscal adjustments ($TB_t = 1$) between 1978-1981 and 1985-1988; ii. there are three periods of EB fiscal adjustments ($EB_t = 1$) between 1990-1992, 1993-1998 and 2011-2013.

To summarize, we classify fiscal consolidations into TB and EB fiscal adjustment plans to account for the observed correlation between tax and expenditure adjustments. This correlation comes from the fact that policy makers implement fiscal consolidations by adopting multi-year fiscal plans with both tax hikes and spending cuts.

Finally, we highlight that fiscal consolidations censor changes in G and T above and below 0, respectively, by construction.⁸ Therefore, estimates of

⁸We thank Valerie Ramey for bringing up this point.

their economic effects are valid for this type of fiscal policy only. Simply put, we do not estimate tax and government spending multipliers.⁹ However, if the United States plans to undertake either a TB or an EB fiscal consolidation, our estimates are externally valid and can be used as a benchmark for policy-makers.

2.2 Aggregate Effects of Fiscal Consolidations in the US

The first step of our analysis is to study the aggregate effects of fiscal consolidations in the US. We estimate the impulse response functions of EB and TB plans using a truncated moving average (MA) representation as in C. D. Romer and D. H. Romer (2010) but where the shocks are given by the fiscal consolidations as in Alesina, Favero, and Giavazzi (2015). So we simulate the response to an unanticipated component taking into account that it is accompanied by the announcement of future changes. Following Alesina, Favero, and Giavazzi (2015) we compute impulse response functions as a difference between the forecast obtained conditionally on a fiscal adjustment plan and the forecast with no plan.¹⁰ Figure 3 shows the estimated cumulative impulse response function of output and employment growth rates using quarterly data from 1978Q1 to 2014Q4.

The left panel of Figure 3 shows that TB plans trigger a cumulative drop of output and employment by 4% and 2% respectively. On the contrary, EB plans do not seem to be recessionary. This result is in line with the findings of the fiscal policy literature.

We repeat the analysis on each component of GDP and report the estimated cumulative impulse response functions in Figure 4.

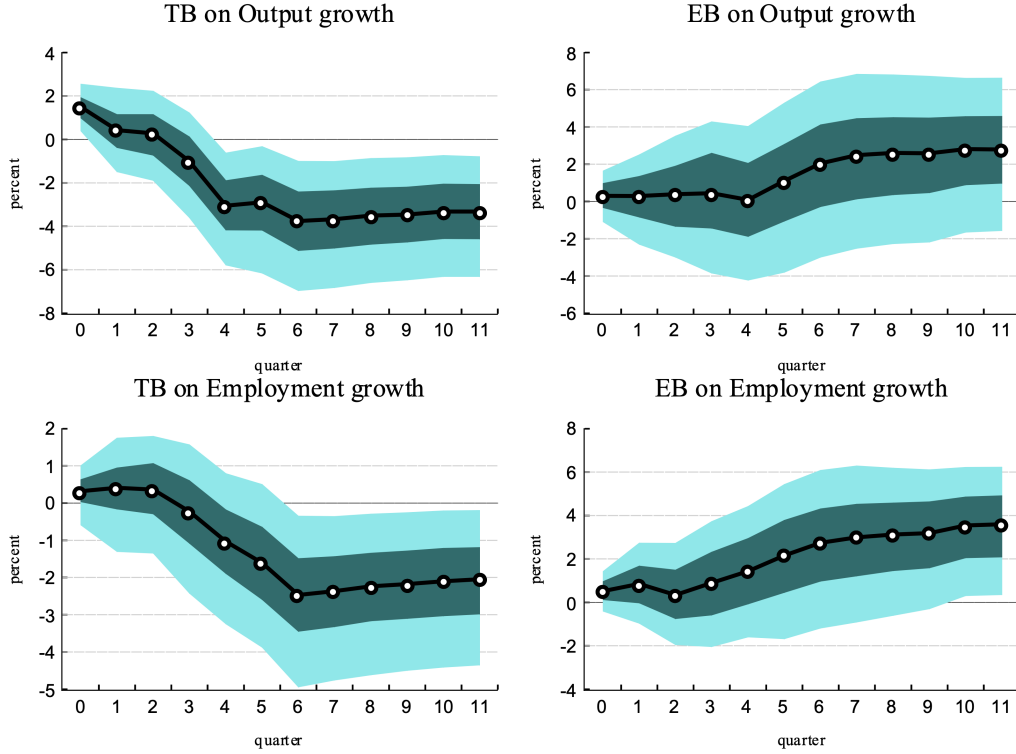
We find that TB fiscal plans are associated with lower than average consumption growth while the other components of GDP do not respond. On the contrary, EB fiscal consolidations exhibit increases in each component of private GDP which are not statistically significant while government spending falls significantly.

Having illustrated what happens to all components of output we turn our attention on the type of fiscal policy change implemented during years of austerity. Firstly, we estimate the effects of fiscal plans on the growth rates of

⁹Unlike us Zubairy (2014) uses a dynamic stochastic general equilibrium framework to investigate the transmission mechanism of fiscal multipliers. Moreover, Evans, Honkapohja, and Mitra (2022) show that spending multiplier depends both on the current state of expectations, as well as the size and duration of the expenditure increase.

¹⁰See Appendix A for details on the data and the regression equation.

Figure 3: Output and Employment Response to Austerity

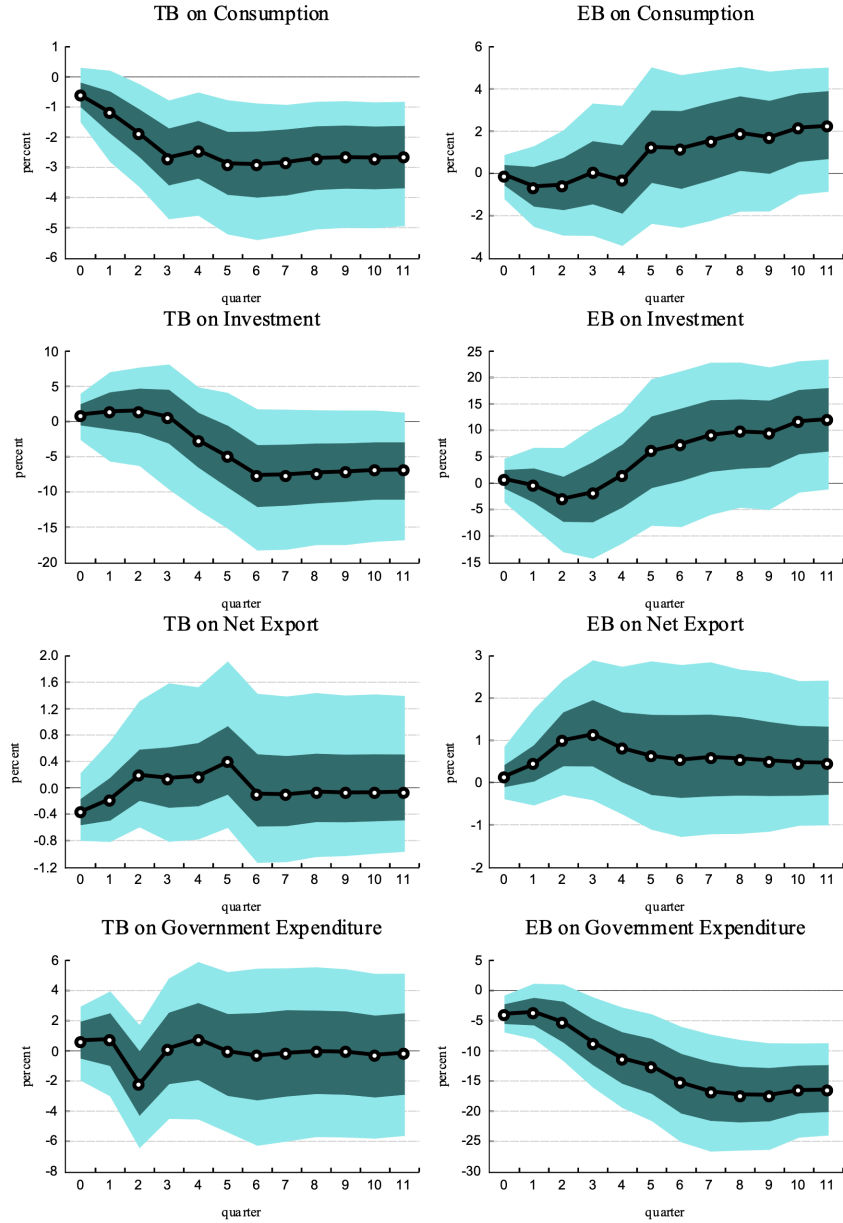


Cumulative IRFs of Output and Employment growth. The darker region refers to the 68% confidence level while the lighter region represents the 95% confidence level, obtained via residual block-bootstrap. Quarterly data. Sample goes from 1978Q1 to 2014Q4. See Appendix A for further details on estimation equation.

government receipts shares of output using again a truncated moving average. Figure 5 shows the cumulative response of government receipts shares of output coming from excise/production taxes and payroll taxes. Other types of government receipts such as corporate tax, income tax, estate/gift tax and custom duties are not affected (see Appendix A).

Looking at the top-left panel, we find that excise/production taxes are the main component of government receipt affected by TB fiscal consolidations, increasing its share of GDP by 12% over a three-year horizon. On the contrary, the change in excise taxes share of output during EB fiscal plans is not statistically different from zero (see top-right panel). This goes in line with Dabla-Norris and Lima (2023) who by investigating the macroeconomic consequences of tax changes during fiscal consolidation periods-distinguished by changes in tax rates and tax base modifications-reveals that base broadening

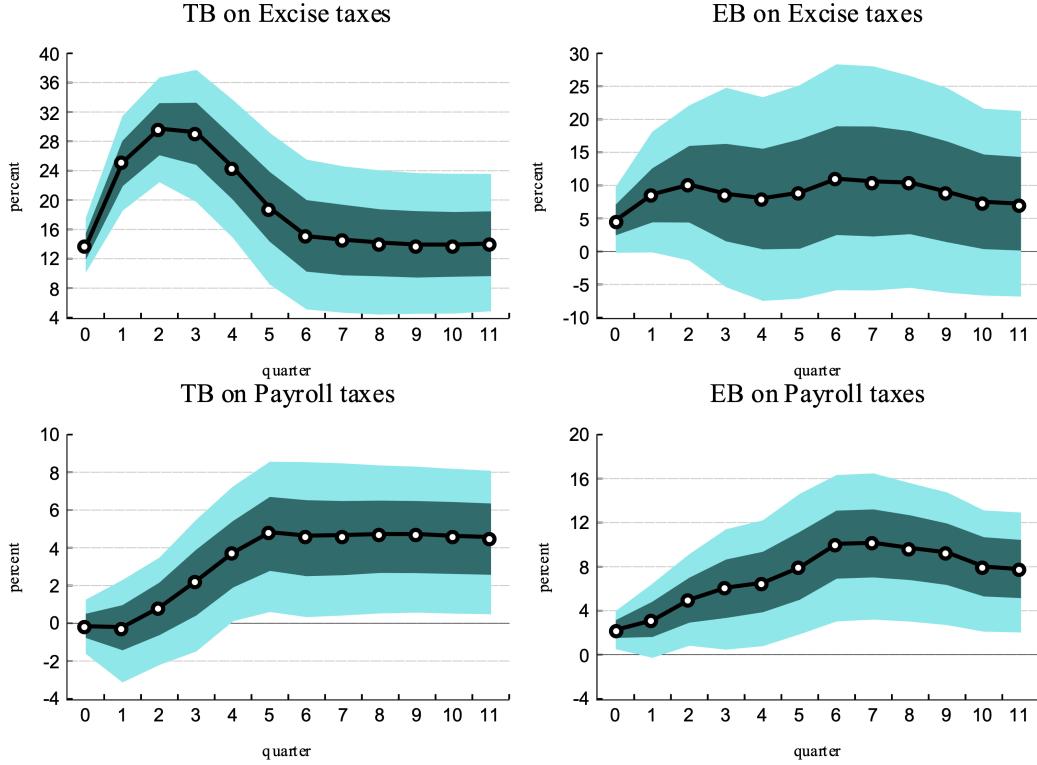
Figure 4: GDP by Austerity



Variables are in real dollars (source NIPA). The darker region refers to the 68% confidence level while the lighter region represents the 95% confidence level obtained via block-bootstrap. Quarterly data. Sample goes from 1978Q1 to 2014Q4.

measures lead to less severe declines in output and employment than rate increases. Interestingly, they show that consolidations heavily rely on increases

Figure 5: Government Revenues Affected by Austerity



Cumulative IRFs of government receipts share of GDP. The darker region refers to the 68% confidence level while the lighter region represents the 95% confidence level, obtained via residual block-bootstrap as suggested in Jentsch and Lunsford (2019a). Quarterly data. Sample goes from 1978Q1 to 2014Q4.

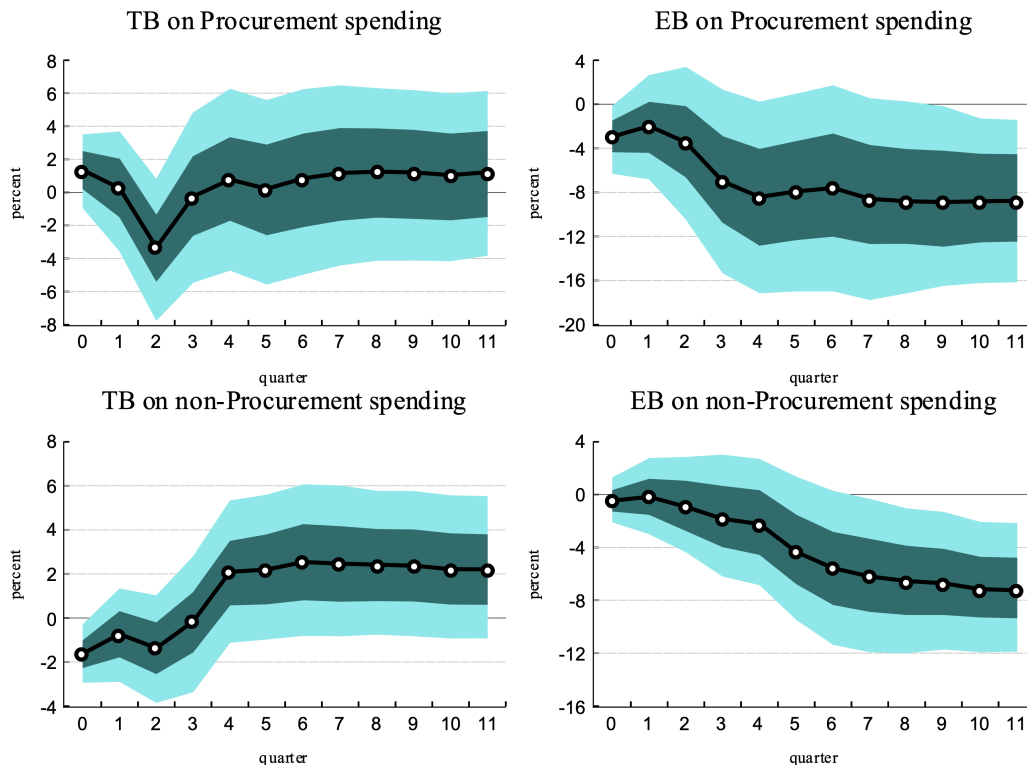
in indirect taxes (both VAT and excises) which both lean towards an emphasis on increasing tax rates and not base changes. Looking at the bottom panels, government receipts share of output coming from payroll taxes appears to increase during both TB and EB fiscal consolidations. This is not surprising if we look back at Figure 2: EB plans also have a small fraction of tax increases in their style (i.e. intra-temporal correlation).

Secondly, we study what happens to government expenditures during years of fiscal consolidations. In particular, we break down government spending, G , into two components: procurement spending and the residual part of G , non-procurement spending.¹¹ Figure 6 shows the cumulative impulse response function of the two components of G as share of GDP. Notice from the right column that only EB plans affect government expenditures. In fact, recall

¹¹We measure procurement spending as done in Briganti and Sellemi (2022).

from Figure 2 that TB plans are pure tax hikes.

Figure 6: Government Expenditures Affected by Austerity



Cumulative IRFs of government expenditure two components shares of GDP. The darker region refers to the 68% confidence level while the lighter region represents the 95% confidence level, obtained via residual block-bootstrap as suggested in Jentsch and Lunsford (2019a). Quarterly data. Sample goes from 1978Q1 to 2014Q4.

Overall, the aggregate results show that tax-based austerity plans (i) were recessionary, (ii) hit especially consumption and (iii) were implemented by increasing payroll and excise taxes. The expenditure side was unaffected. On the contrary, spending-cuts austerity was characterized by mild and statistically insignificant increases in output and was mainly done via equal cuts in procurement spending and the rest of government consumption expenditure.¹²

¹²EB plans also increase payroll taxes, however, they account for a minor part of the total EB plan (see again Figure 2).

2.3 Fiscal Plans and Production Networks

In the previous section we highlight an important difference between TB and EB plans: positive changes in excise/production taxes are unique to TB plans while procurement spending cuts are unique to EB fiscal consolidations.

In this section we use Acemoglu, Akcigit, and Kerr (2016)'s framework to convey intuition and illustrate different propagation mechanisms of taxes and government purchases in a standard static multi-sector Neoclassical model. In this framework a change in government purchases behaves as a demand shock which propagates upstream in a production network: an industry affected by a demand shock propagates the shock to all its suppliers of input. On the contrary, excise/production taxes behave as supply shocks which propagate downstream: an industry affected by a supply shock passes the shock to all its customers. Step by step, the shock trickles down to consumers. Notice that this asymmetric transmission mechanism of taxes and government purchases is consistent with the asymmetric response of consumption during years of TB and EB fiscal consolidations.

In this section we explore the theoretical propagation of these types of fiscal policy through the lens of a simple static model with production network, which is a slightly modified version of Acemoglu, Akcigit, and Kerr (2016).¹³

In this model the economy is inhabited by a representative agent with Cobb-Douglas utility over n -goods. On the production side, the representative sector i maximizes profits:

$$\max_{l_i, \{x_{ij}\}_{j=1}^n} (1 - \tau) \cdot p_i \cdot \underbrace{\left(l_i^{\alpha_i^l} \cdot \left(\prod_{j=1}^n x_{ij}^{a_{ij}} \right)^\rho \right)}_{:=y_i} - w l_i - \sum_{j=1}^n p_j x_{ij}$$

where τ is the excise/production tax, p_i is the price of output i , l_i is the labor input of sector i , $x_{i,j}$ is the quantity of intermediate good j purchased by sector i as input of production, w is the wage and y_i is output of producer of good i .¹⁴ The resource constraint of the economy is:

$$y_i = c_i + \sum_{j=1}^n x_{ji} + G_i$$

where c_i and G_i are consumption and government purchases of good i respectively, while x_{ji} is the quantity of good i used as input of production by sector j .

¹³The modifications come from the inclusion of a production tax and an extra parameter in the production function. We remand to the Appendix E for the detailed derivations.

¹⁴Because of constant return to scale we have $\alpha_i^l + \rho \cdot \sum_{j=1}^n a_{i,j} = 1$ for all sectors.

j .

EB Plans

In this static economy a change in government purchases has the following output effect.¹⁵

$$d \log y_i = \rho \cdot \sum_{j=1}^n a_{ji} \cdot \underbrace{\frac{p_j \cdot y_j}{p_i \cdot y_i}}_{:= \hat{a}_{ji}} \cdot d \log y_j + \frac{G_i}{y_i} \cdot d \log G_i, \quad (2)$$

which in matrix form becomes:

$$d \log \mathbf{y} = \rho \cdot \hat{A}^T \cdot d \log \mathbf{y} + \Lambda \cdot d \log \mathbf{G}$$

$n \times 1$

where $\Lambda = \text{diag}(G_1/y_1, \dots, G_n/y_n)$ and $\hat{A} = [a_{ji} \cdot (p_j \cdot y_j)/(p_i \cdot y_i)]_{i,j=1,\dots,n}$. Moreover, in equilibrium, we also have:

$$\hat{A}^T_{n \times n} \propto \left[\frac{p_i \cdot x_{ji}}{p_i \cdot y_i} \right]_{i,j=1,\dots,n} = \left[\frac{\text{SALES}_{i \rightarrow j}}{\text{OUTPUT}_i} \right]_{i,j=1,\dots,n}$$

The $i - j$ element of \hat{A}^T is proportional to the sales of sector i to sector j , relative to its output, y_i . Therefore, the transmission of government purchases works from customers (sector j) to suppliers (sector i). Finally, to understand the transmission mechanism of government purchases, it is convenient to solve the above expression and then expand it using the definition of geometric sum:

$$\begin{aligned} d \log \mathbf{y} &= \left(I_n - \rho \cdot \hat{A}^T \right)^{-1} \cdot \Lambda \cdot d \log \mathbf{G} \\ &= \left(I_n + \rho \cdot \hat{A}^T + \rho^2 \cdot (\hat{A}^T)^2 + \dots \right) \cdot \Lambda \cdot d \log \mathbf{G} \end{aligned} \quad (3)$$

Equation (3) is saying that spending cuts propagate upstream in the production network. For example, consider a spending cut on good j (i.e. $d \log G_j < 0$). Firstly, output is directly reduced and this first order effect is represented by matrix I_n in the geometric sum expansion. Secondly, sector j reduces the amount of input it needs. Therefore, for each sector i , supplier of j , we have that $x_{ji} = \text{SALES}_{i \rightarrow j}$ decreases. This is a second order effect working via $\rho \cdot \hat{A}^T$. Thirdly, suppliers of suppliers of producer of good j also face an indirect effect and so on and so forth. Since the propagation of the spending cut happens from customers to suppliers, we refer to this type of transmission mechanism as *upstream propagation*.

¹⁵See Appendix E.

Example: 3 Sectors Economy. We further clarify this type of propagation using a simple numerical example illustrated in Figure 7. In the example we

Figure 7: Example of Spending Cut

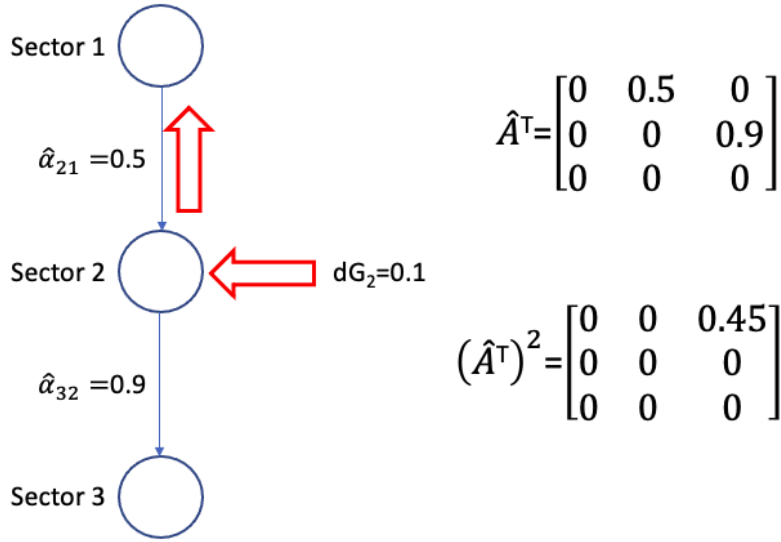


Figure 7: vertically integrated 3 sectors economy. $\hat{a}_{21} = 0.5$ means that sector 1 sells 50% of its output to sector 2. $\hat{a}_{32} = 0.9$ means that sector 2 sells 90% of its output to sector 3. $\hat{a}_{31}^2 := \hat{a}_{21} \cdot \hat{a}_{32} = 0.45$, it means that 45% of sector's 1 output is indirectly purchased by sector 3 via sector 2.

have an economy with three sectors which are vertically integrated: sector 1 supplies sector 2 which supplies sector 3. The *upstream* input-output matrix is given by \hat{A}^T , which is sparse everywhere but in positions 1-2 and 2-3, which reflect the fact that sector 1 supplies sector 2 and sector 2 supplies sector 3. Moreover, $(\hat{A}^T)^2$ represents the second order connection, that is, the suppliers of the suppliers. This production network is characterized by a single second order connection: sector 1 indirectly supplies sector 3 via sector 2. In fact, $(\hat{A}^T)^2$ is sparse everywhere but in position 1-3.

Suppose the government cuts by 0.1 the demand from sector 2. Suppose also that government spending shares of sectoral output are all the same before the policy change, then, the output effect implied by Equation (3) is:

$$d \log \mathbf{y} = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \rho \begin{bmatrix} 0 & 0.5 & 0 \\ 0 & 0 & 0.9 \\ 0 & 0 & 0 \end{bmatrix} + \rho^2 \begin{bmatrix} 0 & 0 & 0.45 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) \cdot \begin{bmatrix} 0 \\ -0.1 \\ 0 \end{bmatrix} = - \begin{bmatrix} 0.05 \cdot \rho \\ 0.1 \\ 0 \end{bmatrix}$$

Notice that sector 2 is hit directly by the shock and its output shrinks exactly by 0.1. Afterwards, the shock travels upstream in the production network, hitting sector 1 because it is the input-supplier of sector 2. On the contrary sectors located downstream in the network are not affected. Finally, the aggregate output effect is given by the average of the sectoral output changes: $d \log y = 1/3 \cdot (0.1 + \rho \cdot 0.05)$. Notice that the stronger the intensity of the input-output connections, represented by ρ , the stronger the aggregate output effect.

Therefore, the model suggests that during years of EB fiscal consolidations sectors located upstream in the production network should be negatively affected by input-output spillovers coming from those cuts in government purchases which we documented in the previous section. Moreover, the total output effect and the network-effect are proportional to the intensity of the upstream propagation during those years, represented in the model by ρ .

TB Plans

When the government increases the production/excise tax, the model returns the following output change:

$$d \log y_i = \rho \cdot \sum_{j=1}^n a_{ij} \cdot d \log y_j - \psi_i \cdot d \log \tau_i, \quad (4)$$

where $\psi_i > 0$.¹⁶ In matrix form the above expression becomes:

$$d \log \mathbf{y}_{n \times 1} = \rho \cdot A \cdot d \log \mathbf{y} - \Psi \cdot d \log \boldsymbol{\tau}$$

where $\Psi = \text{diag}(\psi_1, \dots, \psi_n)$ and $A = [a_{ij}]_{i,j=1,\dots,n}$. The economic interpretation of A is the opposite of the one of \hat{A}^T . In fact, in equilibrium we have:

$$A_{n \times n} \propto \left[\frac{p_j \cdot x_{ij}}{p_i \cdot y_i} \right]_{i,j=1,\dots,n} = \left[\frac{\text{SALES}_{j \rightarrow i}}{\text{OUTPUT}_i} \right]_{i,j=1,\dots,n}$$

that is, the $i - j$ element is proportional to the purchase of good j by sector i relative to its output, y_i . In this case, the transmission mechanism works from suppliers (sector j) to customers (sector i).

Once again, we solve the above expression and then expand it using the definition of geometric sum:

$$d \log \mathbf{y}_{n \times 1} = - (I_n + \rho \cdot A + \rho^2 \cdot A^2 + \dots) \cdot \Psi \cdot d \log \boldsymbol{\tau} \quad (5)$$

¹⁶See Appendix E for derivation.

In this case, the shock propagates in the production network from suppliers to customers via matrix A . In particular, the underlying transmission of the shock works through price increases. In fact, in equilibrium we have:

$$d \log p_i = \rho \cdot \sum_{j=1}^n a_{ij} \cdot d \log p_j + \frac{\tau_i}{1 - \tau_i} \cdot d \log \tau_i.$$

When taxes increase, production becomes more costly and prices go up. A price increase impact the direct customers of the taxed producer, which react by increasing their price too. Eventually, the price-increase trickles down to consumers who respond by decreasing consumption:

$$d \log c_i = -d \log p_i.$$

Therefore, we refer to this type of transmission mechanism as *downstream propagation*.

Consider the example of the vertically integrated 3-sectors economy of Figure 7. In this case, a tax specific shock to sector 2, would have a direct effect on sector 2. Secondly, the shock would travel downstream in the production network via matrix A and would hit sector 3. The tax increase hits the supplier (sector 2), which increases prices, thus damaging its customer (sector 3). On the contrary, sector 1, located upstream in the supply chain, would not be affected by the tax shock.

Therefore, the model suggests that during years of TB fiscal consolidations, when excise/production taxes were increased, sectors located downstream, as well as consumers, are hit by negative spillovers from sectors located upstream in the production network.

3 The Network Effect of Fiscal Plans: Results

In the previous section we illustrated that fiscal policy changes implemented during years of fiscal consolidations, namely excise/production tax increases and procurement spending cuts, propagate downstream and upstream in the production network. In particular, expression (2) suggests that changes in sectoral output, $d \log \mathbf{y}$, during years of EB fiscal adjustment plans, should be proportional to an *upstream spatial lag*, $\rho \cdot \hat{A}^T \cdot d \log \mathbf{y}$ and the spending shock, that is, the EB fiscal adjustment plan. Similarly, expression (4) suggests that changes in sectoral output during years of TB fiscal adjustment plans, should be proportional to a *downstream spatial lag*, $\rho \cdot A \cdot d \log \mathbf{y}$ and the tax shock, that is, the TB fiscal adjustment plan.

Therefore, the most natural regression equation to test the intensity of the propagation of fiscal consolidations in the production network is:¹⁷

$$\begin{aligned} \Delta \log y_{i,t} = & a_i + \left(\underbrace{\rho^{down} \cdot \sum_{j=1}^n a_{ij} \cdot \Delta \log y_{j,t}}_{\Delta y_{i,t}^{down}} + \psi_i \cdot \underbrace{(\tau_u \cdot f_t^u + \tau_a \cdot f_t^a + \tau^f \cdot f_t^f)}_{\text{Tax Increases}} \right) \cdot TB_t + \\ & + \left(\underbrace{\rho^{up} \cdot \sum_{j=1}^n \hat{a}_{ji} \cdot \Delta \log y_{j,t}}_{\Delta y_{i,t}^{up}} + \lambda_i \cdot \underbrace{(\gamma_u \cdot f_t^u + \gamma_a \cdot f_t^a + \gamma^f \cdot f_t^f)}_{\text{Spending Cuts}} \right) \cdot EB_t + \nu_{i,t} \end{aligned} \quad (6)$$

Firstly, Equation (6) includes industry fixed effects, a_i , industry weights ψ_i and λ_i for TB and EB fiscal consolidations respectively and $\nu_{i,t}$, a serially uncorrelated, heteroskedastic error term. We allow for heteroskedasticity since sectors exhibit different volatility in growth rates in the data. Secondly, the first line of Equation (6) contains the downstream spatial variable which captures the downstream spillovers of the unanticipated, announced and future tax increases. Both are interacted with TB_t , the dummy variable which is one during years of TB fiscal consolidations. Similarly, the second line of Equation (6) captures the effects of EB fiscal consolidations as well as its upstream spillovers.

Our econometric specification relates to Alesina, Favero, and Giavazzi (2015), who regress country-level output growth on the 3 components of TB and EB country-specific fiscal plans. Unlike them, we focus on a single country, the United States, by breaking down its economy into $n = 62$ industries. Furthermore, we enrich their specification with two *spatial variables* to take into account the input-output connections among sectors and break down the output effect into a direct and a network effects. This is similar to the empirical approach in Acemoglu, Akcigit, and Kerr (2016) and Ozdagli and Weber (2017).

¹⁷We denote the parameters that we estimate in blue.

3.1 Model Estimation

We focus on a partition of the US economy made by 62 industries, observed from 1978 to 2014 at a yearly frequency. Details on the data construction are reported in Appendix B.

We report results based on the static spatial panel autoregressive models specified by Equation (6). The spatial models allow us to track the effect of EB and TB fiscal adjustment plans on industry output growth, while controlling for downstream and upstream spillovers. When estimating the corresponding parameters, standard OLS delivers inconsistent results since the spatial variables are endogenous. We overcome this problem using spatial econometric techniques. In particular, we use a modified version of the Bayesian Markov Chain Monte Carlo (MCMC) illustrated in LeSage and Pace (2009) to estimate the parameters of equation (6). We also report Maximum Likelihood Estimates (MLE) for two main reasons: (i) if all priors are non-informative, then the Bayesian MCMC should exactly return the MLE, (ii) MLE properties of spatial panel autoregressive models with fixed effects are well known (see J. Yu, DeJong, and Lee (2008)).¹⁸ The derivation of the Bayesian MCMC and of the MLE as well as other technical details are remanded to Appendix C.3.

Table II reports descriptive statistics of the estimated parameters of interest of model (6):

Firstly, looking at Table II, we notice that the maximum likelihood estimates are very close to the expected value and standard deviation of the posterior distributions estimated by MCMC. This is a consequence of using mainly non-informative priors. Secondly, we notice that during years of TB fiscal consolidations, the downstream spatial correlation is much stronger than the upstream spatial correlation during EB fiscal consolidations. In fact, looking at the quantiles of the posterior distribution of ρ^{up} , it is clear that it is much more skewed towards zero than then one of ρ^{down} , and with a posterior average of 0.25 against 0.57 of ρ^{down} .

Concerning the fiscal coefficients, we find that announced tax rises, τ_a , and future spending cuts, γ_f , exhibit a statistical significant recessionary effect, while the other shocks do not. Their posterior probability of being negative is 92% and 97% respectively. Interestingly, the effect of announced spending cuts, γ_a , is statistically significant and expansionary, or positive. Nevertheless, the single coefficients of the three components of fiscal adjustment plans are not very informative: we are interested in the convex combi-

¹⁸Bayesian MCMC is also more appealing than MLE for some quite technical reasons. However, we save these details for Appendix C.3. An alternative approach can be a generalized moments estimator offered by Kelejian and Prucha (1999).

Table II: Estimation Results

<i>Baseline Model - Equation (6)</i>												
<i>Parameters</i>	MLE		Bayesian MCMC - Posterior Distributions:									
	$\hat{\theta}_i^{ML}$	MLE Std.	$\mathbb{E}(\theta_i)$	$\sqrt{\mathbb{V}(\theta_i)}$	$Pr(\theta_i < 0)$	5%	10%	16%	50%	84%	90%	95%
ρ^{down} (TB)	0.603	0.125	0.569	0.117	0.000	0.374	0.419	0.453	0.569	0.687	0.720	0.761
τ_u	0.411	1.278	0.555	1.196	0.322	-1.411	-0.971	-0.629	0.551	1.743	2.095	2.533
τ_a	-1.259	0.990	-1.294	0.930	0.917	-2.820	-2.488	-2.218	-1.295	-0.366	-0.100	0.237
τ_f	-0.192	0.432	-0.219	0.404	0.708	-0.887	-0.735	-0.621	-0.220	0.182	0.300	0.447
ρ^{up} (EB)	0.271	0.092	0.247	0.096	0.000	0.088	0.121	0.148	0.246	0.343	0.372	0.407
γ_u	-0.167	1.129	-0.132	1.046	0.551	-1.855	-1.460	-1.166	-0.130	0.907	1.207	1.582
γ_a	0.942	0.616	1.037	0.582	0.038	0.077	0.292	0.461	1.039	1.610	1.779	1.997
γ_f	-0.477	0.283	-0.482	0.261	0.968	-0.908	-0.817	-0.742	-0.481	-0.224	-0.148	-0.053
D2008	-2.941	0.671	-2.903	0.633	1.000	-3.946	-3.714	-3.532	-2.902	-2.274	-2.092	-1.861
D2009	-5.664	0.671	-5.326	0.658	1.000	-6.416	-6.173	-5.981	-5.321	-4.672	-4.488	-4.248

Table II: θ_i denotes a generic parameter that we estimate. The columns report the following: $\hat{\theta}_i^{ML}$ is the ML point estimate; “MLE Std.” is the standard deviation of the ML estimate, calculated using the analytical Fisher Information Matrix derived in Appendix C.2: $\sqrt{\mathcal{J}(\hat{\theta}_i^{ML})^{-1}}$; $\mathbb{E}(\theta_i)$ is the expected value of the posterior distribution; $\sqrt{\mathbb{V}(\theta_i)}$ is the standard deviation of the posterior distribution; $Pr(\theta < 0)$ is the probability that a parameter is negative, calculated by integrating the posterior distribution; $p\%$ is the p -th percentile of the posterior distribution. For brevity we don’t report here the Industry Fixed Effects and the Industry specific variances. We also include year dummies for 2008 and 2009 to improve the precision of our estimates by capturing the industry-wide dip caused by the Great Recession. In the first columns, the spatial parameters also report the type of fiscal plan they are interacted with (in blue).

nation of all three components in a fiscal plan. Similarly, the mere size of the spatial coefficients is not enough to quantify the aggregate direct and network effect. We address these issues in the following section.

3.2 Aggregate Output Effect of Fiscal Consolidations

We are interested in estimating the average aggregate output effect of fiscal consolidations and then breaking it down into its direct and network effect. Our spatial econometric methodology conveniently provides such a decomposition.

Firstly, fiscal consolidations are made of three components: unanticipated, anticipated, and future. Therefore, we cannot define the impulse response in the standard way as the the partial derivative of a dependent variable with respect to a single shock. Rather, we construct the impulse response as a convex combination of the individual derivatives of $\Delta \log \mathbf{y}_t$ with respect to each of the three components of fiscal consolidations. The weights on each component are determined by the “style” of the plan, defined analytically as:

$$\underbrace{\mathbf{s}_{TB}}_{3 \times 1} := \begin{bmatrix} s_{TB}^u & s_{TB}^a & s_{TB}^f \end{bmatrix}^T \quad \underbrace{\mathbf{s}_{EB}}_{3 \times 1} := \begin{bmatrix} s_{EB}^u & s_{EB}^a & s_{EB}^f \end{bmatrix}^T.$$

For instance, if we want to simulate the effects of a TB fiscal plan which is 30% unanticipated, 0% anticipated, and 70% future, then we would set: $s_{TB}^u = .3$,

$s_{TB}^a = 0$, $s_{TB}^f = .7$ and the vector of the “style” would be: $\mathbf{s}_{TB} = [.3 \ 0 \ .7]^T$.

Secondly, given: *i.* the above definition of impulse response, *ii.* the vector representation of Equation (6), *iii.* the vectors of fiscal parameters $\boldsymbol{\tau}^T = [\tau_u \ \tau_a \ \tau_f]$ and $\boldsymbol{\gamma}^T = [\gamma_u \ \gamma_a \ \gamma_f]$ and *iv.* industry weights for TB plans $\boldsymbol{\Psi}^T = [\psi_1 \dots \psi_n]$ and EB plans $\boldsymbol{\Lambda}^T = [\lambda_1 \dots \lambda_n]$; then, the $n \times 1$ vector of industry specific Total Effect of a TB plan ($TB_t = 1$ and $EB_t = 0$) is defined as:

$$\begin{aligned} TE_{TB} &:= s_{TB}^u \cdot \left. \frac{\partial \Delta \log \mathbf{y}_t}{\partial f_t^u} \right|_{TB_t=1} + s_{TB}^a \cdot \left. \frac{\partial \Delta \log \mathbf{y}_t}{\partial f_t^a} \right|_{TB_t=1} + s_{TB}^f \cdot \left. \frac{\partial \Delta \log \mathbf{y}_t}{\partial f_t^f} \right|_{TB_t=1} \\ &= \underbrace{(I_n - \rho^{down} \cdot A)^{-1}}_{:= \mathbf{H}^{TB}} \cdot \boldsymbol{\Psi} \cdot \boldsymbol{\tau}^T \cdot \mathbf{s}_{TB} = \underbrace{\mathbf{H}^{TB} \cdot \boldsymbol{\Psi}}_{n \times 1} \cdot \underbrace{\boldsymbol{\tau}^T \cdot \mathbf{s}_{TB}}_{1 \times 1} \end{aligned}$$

Analogously, for an EB plan we have:

$$TE_{EB} := \underbrace{(I_n - \rho^{up} \cdot \hat{A}_0^T)^{-1}}_{:= \mathbf{H}^{EB}} \cdot \boldsymbol{\Lambda} \cdot \boldsymbol{\gamma}^T \cdot \mathbf{s}_{EB} = \underbrace{\mathbf{H}^{EB} \cdot \boldsymbol{\Lambda}}_{n \times 1} \cdot \underbrace{\boldsymbol{\gamma}^T \cdot \mathbf{s}_{EB}}_{1 \times 1}.$$

Using the spatial framework, we can break down the TE into a Direct and Network Effect, as in Acemoglu, Akcigit, and Kerr (2016) and Ozdagli and Weber (2017). The former represents the direct impact of the fiscal plan and the latter represents the network spillovers:

$$\begin{aligned} DE_{TB} &= \boldsymbol{\Psi} \cdot \boldsymbol{\tau}^T \cdot \mathbf{s}_{TB} & NE_{TB} &= (\mathbf{H}^{TB} - I_n) \cdot \boldsymbol{\Psi} \cdot \boldsymbol{\tau}^T \cdot \mathbf{s}_{TB} \\ DE_{EB} &= \boldsymbol{\Lambda} \cdot \boldsymbol{\gamma}^T \cdot \mathbf{s}_{EB} & NE_{EB} &= (\mathbf{H}^{EB} - I_n) \cdot \boldsymbol{\Lambda} \cdot \boldsymbol{\gamma}^T \cdot \mathbf{s}_{EB}. \end{aligned}$$

The TE, DE and NE are $n \times 1$ vectors of industry specific effects of fiscal adjustment plans. However, we are interested in their aggregate effect. Therefore, we take a weighted average across industries with weights given by each industry’s output share.¹⁹ By doing so we obtain the Average Total Effect, ATE , of a fiscal consolidation. We similarly construct the Average Direct Effect, ADE , and the Average Network Effect, ANE . Notice that given the linearity of the weighted average operation, we have that $ATE = ADE + ANE$, which therefore summarizes the breakdown of the total effect into its two components.

Table III reports descriptive statistics of the posterior distributions of the ATE and its decomposition into ADE and ANE for 2-year fiscal adjustment

¹⁹We use average output shares in years of TB fiscal consolidation for aggregating TB effects. We use average output shares in years of EB fiscal consolidation for aggregating EB effects

plans in the United States. This is our main contribution to the literature on fiscal consolidations. We obtain these results via Monte-Carlo, by drawing the parameters of equation (6) from their estimated posterior distributions.²⁰ The style of the simulated plans, \mathbf{s}_{TB} and \mathbf{s}_{EB} - which determines the composition of a fiscal plan in terms of unanticipated, anticipated, and future components - is randomly drawn at each iteration from a distribution which mimics the in-sample data and satisfies three conditions: 1) the overall size of a plan is 1%; 2) the anticipated component is zero; 3) the horizon of the plan is two years.²¹ This procedure ensures that our results are robust to different styles of fiscal plans and are not driven by a style redistribution of the 1% fiscal shock.

Table III: Average Total, Direct and Network Effects of Fiscal Consolidations in the United States

<i>Baseline Model - Equation (6)</i>											
	$\mathbb{E}(\theta)$	%	$\sqrt{\mathbb{V}(\theta)}$	$Pr(\theta < 0)$	5%	10%	16%	50%	84%	90%	95%
ATE_{TB}	-1.397	100%	1.109	0.904	-3.297	-2.835	-2.487	-1.346	-0.308	-0.027	0.328
ADE_{TB}	-1.017	73%	0.789	0.904	-2.327	-2.031	-1.798	-1.006	-0.238	-0.021	0.258
ANE_{TB}	-0.380	27%	0.337	0.904	-1.014	-0.825	-0.694	-0.328	-0.066	-0.006	0.065
ATE_{EB}	0.370	100%	0.371	0.152	-0.265	-0.103	0.014	0.386	0.727	0.825	0.950
ADE_{EB}	0.326	88%	0.327	0.152	-0.225	-0.088	0.012	0.336	0.643	0.732	0.845
ANE_{EB}	0.043	12%	0.052	0.152	-0.038	-0.014	0.001	0.041	0.090	0.106	0.130

Table III: descriptive statistics of posterior distributions of Average Effects of a 2 years, 1% magnitude fiscal adjustment plan. 2 years means that results are calculated by cumulating the effect of the first year of the plan and then the second one. The style of the plan is simulated from a distribution which mimics the observed one; see Appendix C.3 for technical details. Columns: $\mathbb{E}(\theta)$ is the expected value of the posterior distribution; % is the share of ATE represented by ADE and ANE. $\sqrt{\mathbb{V}(\theta)}$ is the standard deviations of the posterior distribution; $Pr(\theta < 0)$ is the probability of negative values, calculated by integrating the posterior distribution; “p%” is the p-th percentile of the posterior distribution.

In Table III, we document two main facts. First of all, consistent with existing work, TB fiscal consolidations imply larger output losses than EB fiscal consolidations. The expected value of ATE_{TB} is -1.397 against a positive and insignificant ATE_{EB} of 0.370. This implies that a 2 years TB fiscal consolidation of 1% causes a cumulative average contraction of -1.397% over two years. On the other hand, the effects of EB fiscal consolidations are mildly positive and not statistically significant.

Secondly, around 27% of ATE_{TB} comes from network spillovers, confirming the relevance of the industrial network in the transmission of the TB fiscal adjustments. On the contrary, the network propagation of an EB fiscal plan is much

²⁰In doing so we draw all the parameters jointly from each step of the Markov Chain to take into account the potential correlation among the parameters’ distributions.

²¹See Appendix, section C.4, for further information on the empirical distribution of the style of US fiscal plans.

smaller, accounting for only 12% of ATE_{EB} . We calculate the average extent to which differences in the network effects of EB and TB plans account for differences in their total effects: $|\mathbb{E}(ANE_{TB}) - \mathbb{E}(ANE_{EB})| / |\mathbb{E}(ATE_{TB}) - \mathbb{E}(ATE_{EB})|$. We find a value of approximately 25%.²² Therefore, we conclude that at least 25% of the difference between EB and TB output effects can be explained by differences in production network spillovers.

We summarize our findings so far. TB fiscal consolidations have stronger effects in the United States than EB fiscal consolidations, with an average two years contraction of around -1.4%. EB fiscal consolidations in the United States have effects which are either not statistically different from zero, or mildly expansionary after two years.²³ Network effects of TB consolidations explain 27% of the overall contraction. On average, 25% of the differences in the ATE of TB and EB plans can be attributed to the stronger network propagation of TB fiscal consolidations.

4 Robustness

4.1 Spatial Model and Orders of Propagation

An alternative to spatial lags in our econometric model is a standard panel data model with several “cross-terms” representing the first-order, second-order, and higher-order degrees of connection, as in Hale, Kapan, and Minoiu (2019). However, this methodology requires a large number of parameters to be estimated, especially when the network is persistent, and when higher-order propagation effects are relevant. On the contrary, a spatial variable is capable of capturing the entire feedback effect with an infinite number of orders of connection whose impact decays geometrically.

In order to assess whether the US industrial network with $n = 62$ sectors generates relevant high-order spillovers, we perform the partitioning of the effect, similar to what suggested by LeSage and Pace (2009). For instance, for the downstream propagation, we have:

$$\underbrace{(I_n - A)^{-1} \cdot \mathbf{1}_n}_{\text{Total Effect}} = \underbrace{\mathbf{1}_n}_{\text{Direct}} + \underbrace{A \cdot \mathbf{1}_n}_{\text{1st order In-degree}} + \underbrace{A^2 \cdot \mathbf{1}_n}_{\text{2nd order In-degree}} + \dots$$

²²From Table III, we have: $|-0.380 - 0.043| / |-1.397 - 0.370| \approx 25\%$ in the baseline model and $|-0.300 - 0.031| / |-1.148 - 0.522| \approx 20\%$ in the inverted model.

²³This is in line with Alesina, Favero, and Giavazzi (2020). Moreover, Lemoine and Lindé (2016) highlight that the economies with independent monetary policy and high degree of credibility may indeed enjoy expansionary austerity.

where the term in-degree refers to the fact that the row-sum of the elements of A represents the weighted in-degree of the network (total share of input purchased by a sector). For the upstream propagation, we have:

$$\underbrace{(I_n - \hat{A}^T)^{-1} \cdot \mathbf{1}_n}_{\text{Total Effect}} = \underbrace{\mathbf{1}_n}_{\text{Direct}} + \underbrace{\hat{A}^T \cdot \mathbf{1}_n}_{\text{1st order Out-degree}} + \underbrace{(\hat{A}^T)^2 \cdot \mathbf{1}_n}_{\text{2nd order Out-degree}} + \dots$$

where the term out-degree refers to the fact that the row-sum of \hat{A}^T represents the weighted out-degree of the network (total share of output sold to other sectors).²⁴ By averaging across the 62 industries the above expressions, we can calculate how much of the average total effect (left hand side of the expressions) can be attributed to each order of propagation (addends of the right hand side of the expressions). The results are reported in Table IV

Table IV: Partitioning of the network

Order	Downstream Network		Upstream Network	
	%	Cumulative	%	Cumulative
0 (<i>Direct</i>)	53.36%	53.36%	54.53%	54.53%
1 st	24.53%	77.89%	23.34%	77.86%
2 nd	11.49%	89.39%	11.33%	89.20%
3 rd	5.48%	94.87%	5.52%	94.72%
4 th	2.64%	97.51%	2.70%	97.42%
5 th	1.28%	98.79%	1.32%	98.74%
⋮	⋮	⋮	⋮	⋮

Notice that, consistent with Acemoglu, Carvalho, et al. (2012) and Carvalho (2007), the first two orders of the in-degrees and out-degrees are enough to capture most of the spillovers, roughly 89% of the overall effects. However, to capture the whole scope of network effects we should add terms up to the 5th order, which account for almost 99% of the total effect. Since we have 6 “core regressors” (TB and EB unanticipated, announced, and future components), the adoption of cross terms which capture the order of propagation, would require us to include 6 times 5 orders plus one (the Direct effect) for a total of 36 core regressors. Considering this unfeasible econometric specification, we opt for the more parsimonious spatial lag.

²⁴For more on in-degrees and out-degrees of the industrial network see Acemoglu, Carvalho, et al. (2012) and Carvalho and Tahbaz-Salehi (2019).

4.2 Dynamics and Delayed Network Effects

The baseline model specified by Equation (6) does not include any time lag. We adopt a fully static specification because annual industry value-added growth rates are not very persistent, in particular at the fine disaggregation level of 62 sectors. Nevertheless, few sectors still show a non-negligible degree of autocorrelation. Therefore, we check whether our results are robust to the inclusion of a lagged dependent variable and we augment Equation (6) with a time lag: $\phi_i \cdot \Delta \log y_{i,t-1}$. The results are summarized by cumulative dynamic ATE, ADE and ANE, which now take the form of cumulative impulse response functions, reported in Figure 8. The values of the median of the dynamic ATE, ADE and ANE (blue solid lines in Figure 8) are reported in Table V.

Notice that after year 2, the end of the fiscal consolidation, the dynamic

Figure 8: Cumulative Impulse Response Functions

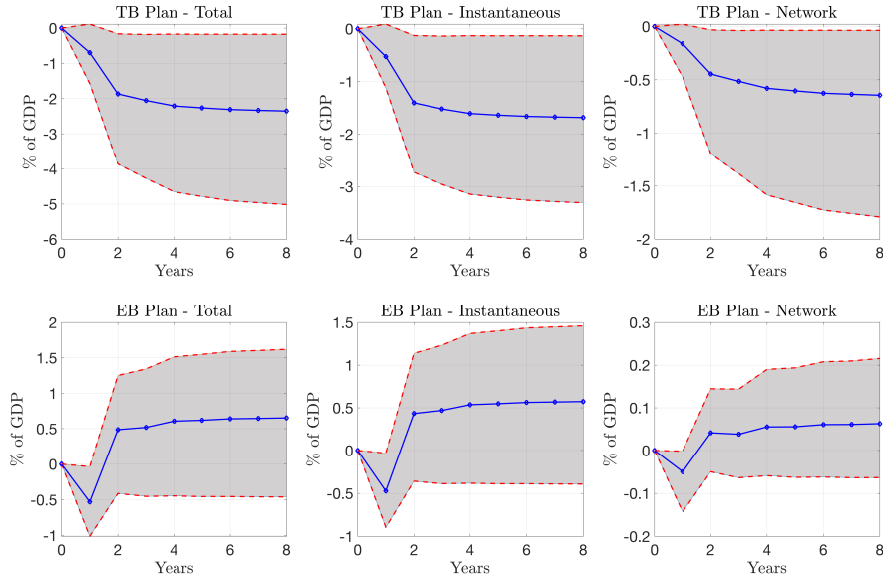


Figure 8: blue solid lines are the median cumulative impulse response functions (median of the posterior distributions). Red dashed lines are the 5th and 95th percentile of their posterior distributions, which represent our confidence bands. The “shock” is constructed by simulating a two years fiscal adjustment plan of 1% of GDP, exactly as done earlier to derive our static baseline results.

response is minimal, which corroborates our static analysis. In general, the effects are slightly larger in year 2 compared to the ones estimated in the static model and reported in Table III. Except for this, the results are comparable: 1)

Table V: Median Cumulative Impulse Response Functions

	1 year	%	2 years	%	...	Long Run	%
ATE_{TB}	-0.695	100%	-1.865	100%	...	-2.351	100%
ADE_{TB}	-0.526	76.7%	-1.403	75.2%	...	-1.683	71.6%
ANE_{TB}	-0.162	23.3%	-0.445	24.8%	...	-0.644	28.4%
ATE_{EB}	-0.523	100%	0.486	100%	...	0.628	100%
ADE_{EB}	-0.472	90.2%	0.433	89.1%	...	0.573	91.2%
ANE_{EB}	-0.049	9.8%	0.041	10.9%	...	0.063	8.8%

TB fiscal consolidations are recessionary and statistically different from zero; 2) the network effect is around one-fourth of the total effect of a TB plan; 3) EB fiscal consolidations have a minor network effect in the order of 10% of the total effect; 4) EB fiscal consolidations seem to be expansionary, but nothing can be concluded since they are not statistically different from zero.

We conclude this section by highlighting one fact: from Table V we notice that the relevance of ANE_{TB} increases over time, from 23.3% to 28.4% in the long-run. This could be indicative of *delayed network effects*. Suppose a price shock takes longer than a year to travel from one sector to another, then the relevance of the network effect will increase over time since the spillover takes time to kick-in. For instance, Smets, Tielens, and Van Hove (2019) show that the autocorrelation between inflation in crude oil’s price and synthetic rubber’s price spikes after three months. Then the autocorrelation between inflation in synthetic rubber’s price and tires’ price also spikes after three months, but the autocorrelation between inflation in tires’ price and transport costs spikes after 16 months. Therefore, downstream propagation of price changes does seem to have delayed effects consistent with the increasing relevance over time of the network effect of TB fiscal adjustments. We leave the issue of timing of the network effect for future research.

4.3 Inverted Propagation Mechanism

The baseline regression equation, Equation (6), implicitly assumes that TB fiscal consolidations exclusively propagate downstream, from suppliers to customers while the opposite is true for EB fiscal consolidations. This is done by interacting TB_t with $\Delta y_{i,t}^{\text{down}}$ and EB_t with $\Delta y_{i,t}^{\text{up}}$. This assumption is consistent with the theoretical propagation suggested by the model.

We now relax the assumption above and we switch the interaction of our dummies with the spatial variables. Therefore, we estimate the following equation:

$$\begin{aligned} \Delta \log y_{i,t} = & \tilde{\alpha}_i + \left(\tilde{\rho}^{down} \cdot \Delta y_{i,t}^{up} + \psi_i \cdot \underbrace{(\tilde{\tau}_u \cdot f_t^u + \tilde{\tau}_a \cdot f_t^a + \tilde{\tau}^f \cdot f_t^f)}_{\text{Tax Increases}} \right) \cdot TB_t + \\ & + \left(\tilde{\rho}^{up} \cdot \Delta y_{i,t}^{down} + \gamma_i \cdot \underbrace{(\tilde{\gamma}_u \cdot f_t^u + \tilde{\gamma}_a \cdot f_t^a + \tilde{\gamma}^f \cdot f_t^f)}_{\text{Spending Cuts}} \right) \cdot EB_t + \tilde{v}_{i,t}. \end{aligned} \quad (7)$$

Another option is to consider all propagation channels at once by estimating a single larger model which nests both Equation (6) (baseline model) and (7) (inverted model). However, this option is intractable due to the large number of parameters relative to the sample size, and due to collinearity between the spatial variables. We therefore estimate two separate models and then we apply a Vuong test for non-nested models to see which one fits the data better (see Vuong (1989) and Wooldridge (2010)). We find that the theoretically consistent model of Equation (6), where TB shocks propagate downstream and EB shocks upstream, provides a better fit to the data but not enough to reject the null hypothesis of the Vuong test, which assumes that the two model describe the data equally well.²⁵

Secondly, we use the new estimates from the inverted model of Equation (7) to calculate the total, direct and network effect.²⁶ We find the network effect of EB plans accounts for only 6% of their total effect, against the 12% of the baseline model. On the contrary, the relevance of network effects of TB plans is basically unaffected, diminishing only by 1% relative to the baseline model (from 27% to 26%). Moreover, its statistical significance declines, since the posterior distribution shrinks towards zero.

Overall, the results indicate that the baseline model, which is consistent with the theoretical transmission channel illustrated in Section 2.3, delivers slightly stronger network effects and a slightly better fit.

4.4 Spurious Correlation and Placebo Experiments

One result of the paper is to record significant network effects of TB fiscal consolidations, accounting for 27% of the total effect, and capable of explaining

²⁵Derivation and details of the Vuong test are outlined in Appendix D.1.

²⁶Tables of results are reported in Appendix D.2.

up to one fourth of the differences between the total output effect of TB and EB fiscal consolidations. What feature of the network is at basis of such strong spillovers? Are we measuring spurious correlation between sectors? or are we capturing some deep structural feature of the industrial network?²⁷

First of all, we plot in Figure 9 the downstream network A associated with the downstream propagation of TB fiscal consolidations. Recall that the generic element of A , denoted by a_{ij} , is given by the reliance of sector i (row) on industrial input j (column): $SALES_{j \rightarrow i} / SALES_i$.

Figure 9: Small, medium and large elements of Downstream Network A

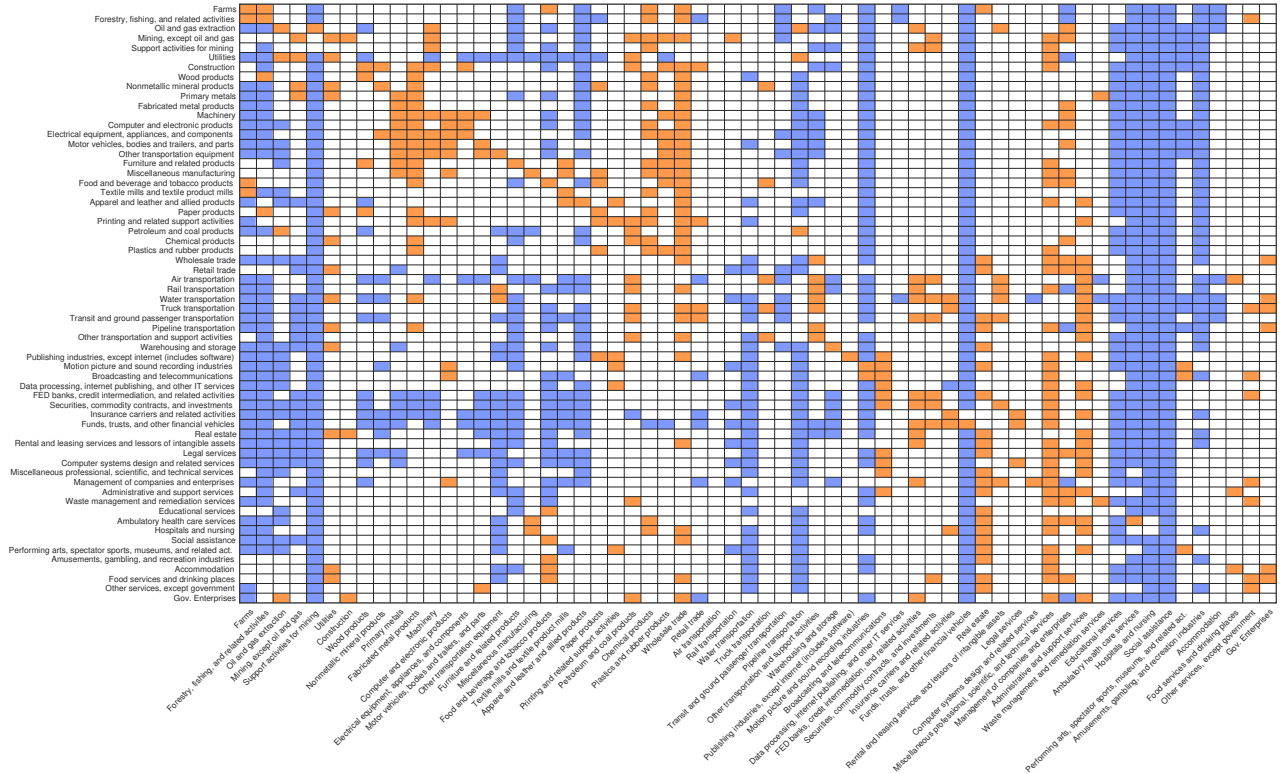


Figure 9 is a “threshold heat-map” which reports a blue cell if $a_{ij} < 0.0001$, an orange cell if $a_{ij} > 0.03$ and a white cell otherwise.²⁸ Two facts are salient

²⁷In investigating the cross-country-sector variation in demand shocks, Joya and Rougier (2019) underscores the crucial role of a sector’s network properties in its capacity to transmit volatility across the economy, emphasizing both contagion and substitution effects. We complement Joya and Rougier (2019) by highlighting the importance of the network structure in propagation of fiscal consolidation within interconnected economic sectors.

²⁸The choice of 0.0001 is motivated by the presence of several values of A which are close

from this “X-ray” of the downstream network. Firstly, the columns of A tend to contain either only very small or only very large values. Secondly, the rows of A do not exhibit such a pattern. In other words, some sectors, such as “Social Assistance” or “Motion Picture and Sound Recording Industries”, produce an output that is either not employed at all as an intermediate by other sectors, or it is employed only in minor quantity. Unlike them, some other sectors, such as “Wholesale Trade” and “Miscellaneous Professional, Scientific and Technical Services”, produce an output which is a key input of production for many sectors. The bottom line is that the US downstream network is characterized by the presence of key suppliers and the lack of key customers. This asymmetric nature of the I-O connections is a well-known feature in the production network literature (see Acemoglu, Carvalho, et al. (2012)).

An interesting robustness exercise is to see what happens to our estimates if we employ simulated network matrices that break this pattern. We estimate Equation (6) (baseline) several times by employing simulated downstream matrices (“placebo”) and compare the results with the original estimates. We carry out two experiments:

- i. *Column-Shuffling*: we randomly shuffle the order of the columns of A and create 100 simulated downstream matrices. This random permutation of the columns allows us to break that natural equilibrium in which some sectors behave as key suppliers and others are marginalized. In fact, in this first simulation, some real-world key supplier might be forced to behave as a peripheral sector and vice-versa. Therefore we expect less statistically significant results.
- ii. *Row-Shuffling*: we randomly shuffle the order of the rows of A . Unlike the first experiment, reshuffling the elements within a column (shuffle the order of the rows) does not break the aforementioned characterizing pattern of the US downstream network. Sectors that originally were key suppliers will still behave in the same way. The same is true for peripheral sectors. We are reshuffling elements with similar magnitude along a column of A . Therefore, we expect to record both stronger and weaker results in terms of statistical significance.

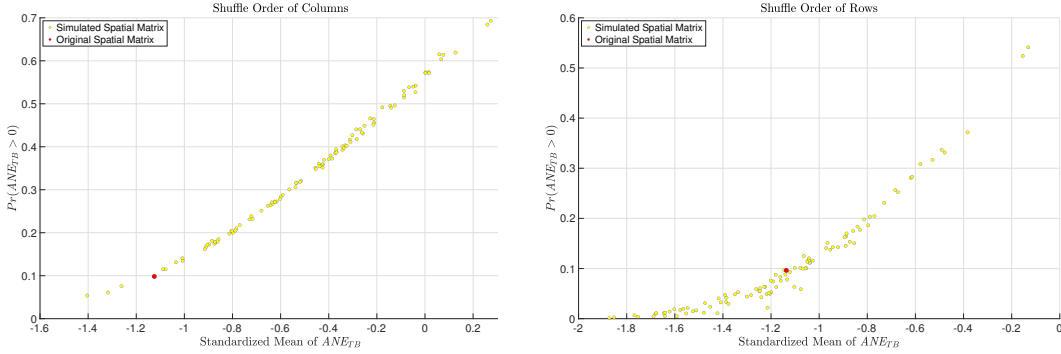
Notice that in a Bayesian framework it is not fully correct to talk about statistical significance, however, with a little abuse of terminology we state that the ANE_{TB} is more statistically significant if the values of $\mathbb{E}(ATE_{TB})/\sqrt{\mathbb{V}(ATE_{TB})}$

to zero but not exactly zero. The choice of 0.03 is motivated by the presence of only a few values above this threshold. In general, tweaking these numbers still allows observing such a visual pattern of matrix A .

and $Pr(ATE_{TB} > 0)$ are both smaller. The first measure represents how many standard deviations we need, to obtain the average ANE_{TB} : the smaller it is, the more likely is to obtain sizable negative spillovers. The second measure is simply the probability of obtaining a non-negative network effect: the smaller it is, the higher the chances of getting recessionary spillovers.

Figure 10 plots on the horizontal axis the first measure and on the vertical axis the second one. The red dot represents the values obtained by employing the original matrix A (see Table III). The left panel of Figure 10 reports the

Figure 10: Placebo Experiment on ANE_{TB}



results of the experiment of shuffling the order of the columns: the red-dot is located in the South-West region of the graph, indicative of more significant spillover effects, as expected. The right panel reports the results of the “row-shuffling” experiment: the red-dot is located almost in the middle of the cloud of simulations’ results, also in line with what expected.²⁹

We highlight that these three steps procedure (simulation of network matrices, re-estimation, and comparison with the original values) is analogous to Ozdagli and Weber (2017). Unlike them, our “placebo” matrices are simulated in a simpler way by simply reshuffling the orders of the columns and rows.

Our procedure has the benefit of preserving the original elements of the network matrices, thus matching one to one both the distribution of the original elements a_{ij} , as well as its sparsity (number of zero entries). Unlike the original network A , the placebo matrices do not have large entries on the main diagonal in either simulations (“dense main diagonal”).

Concerning the first order weighted in-degrees ($A \cdot \mathbf{1}_n$) we have that the placebo

²⁹ Actually, slightly more dots are located more South-West than the original simulation; this is not surprising if we think that we are moving the large elements of the main diagonal (see heat-map 9) outside of it, thus mechanically inflating the indirect spillover of the sector receiving the main diagonal entry.

matrices will exactly match it in the first simulation (shuffling the columns) while in the second one (shuffling the rows), the values are the same but they are assigned to different industries.

The second-order weighted in-degrees ($A^2 \cdot \mathbf{1}_n$) are not matched in either simulation, but the shape of their distribution is similar to the original one. Table VI summarizes the results.

Table VI: Placebo Experiment Results

<i>Network Features:</i>	<i>Shuffling the Columns</i>	<i>Shuffling the Rows</i>
<i>Sparsity</i>	<i>same</i>	<i>same</i>
<i>Distribution of a_{ij}</i>	<i>same</i>	<i>same</i>
<i>Dense Main Diagonal</i>	<i>no</i>	<i>no</i>
<i>1st Weighted In-degree</i>	<i>same values</i>	<i>same distribution</i>
<i>2nd Weighted In-degree</i>	<i>similar distribution</i>	<i>similar distribution</i>
<i>Key Suppliers</i>	<i>same</i>	<i>different</i>
<i>Peripheral Suppliers</i>	<i>same</i>	<i>different</i>
<i>Is original ANE_{TB} stronger?</i>	<i>yes</i>	<i>no</i>

Ozdagli and Weber (2017) conclude that matching the first and second order out-degree is not sufficient to justify the strong upstream propagation of monetary policy shocks. In fact, they say, matching the properties of the network industry by industry is necessary to obtain a strong network effect. We achieve the same conclusion in the context of downstream propagation of TB fiscal consolidations, measured by ANE_{TB} , by means of an easier experiment, namely shuffling the order of rows and columns.

Finally, we answer the initial two questions: the significant downstream network effect of TB fiscal consolidation that we find, is not capturing a spurious relationship between the sectors, otherwise its effects should not be stronger than the placebo ones when we shuffle the columns. In fact, the downstream propagation hinges on the presence of key suppliers of input of production in the industrial network, as witnessed by the lack of superior results when employing the original downstream matrix and we break this pattern (row shuffling).

5 Conclusions

This paper investigates the effects of fiscal consolidations and their propagation in the industrial network in the US from 1978-2014. We find that TB

fiscal consolidations are associated with slower consumption growth and are implemented with excise/production tax increases which are supposed to propagate downstream in the production network, via price increases. EB fiscal consolidations have no recessionary effects and are implemented mainly with procurement spending cuts which propagate upstream in the production network via changes in input-demand. Using a panel of 62 industries, we find evidence of network effects of fiscal consolidations. In particular, we apply spatial econometric techniques to break down the total aggregate effect of fiscal consolidations into a direct component and a network component.

Firstly, we find stronger effects of tax-based fiscal adjustments. In particular, an adjustment of one percent of GDP leads to an average contraction over two years of about -1.4% of value-added. Secondly, 27% of this effect can be attributed to spillovers from a supplying industry to a customer one. Thirdly, we find no evidence for a statistically significant recessionary impact of fiscal consolidations achieved by means of spending cuts. Rather, our evidence indicates mild expansionary effects. Fourthly, only 11% of EB effects originate from an upstream network transmission. Fifthly, we find that almost one-fourth of the different average total effects of TB and EB fiscal consolidations can be explained by stronger network spillovers of the former. Moreover, placebo experiments find that such a network effect of TB fiscal plans originates from the presence of key suppliers in the economy and does not depend on the particular shape of the distribution of first and second-order in-degrees of the network. When those key suppliers are forced to behave as peripheral suppliers the downstream propagation of TB plans vanishes or becomes significantly weaker.

In terms of policy implications, we provide further evidence that a fiscal consolidation based on spending cuts should be preferred to one based on tax hikes. The rationale is that smaller negative spillovers associated with spending cuts reduce the overall output cost. Also, the placebo experiments stress the importance of key suppliers of input in the industrial network. However, we do not comment on the possibility of designing optimal policies which take into account the special role of key suppliers in the propagation of shocks. We plan to address these issues in further research.

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References

- [1] Daron Acemoglu, Ufuk Akcigit, and William Kerr. “Networks and the macroeconomy: An empirical exploration.” In: *NBER Macroeconomics Annual* 30.1 (2016), pp. 273–335. DOI: [10.1086/685961](https://doi.org/10.1086/685961).
- [2] Daron Acemoglu, Vasco Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. “The network origins of aggregate fluctuations.” In: *Econometrica* 80.5 (2012), pp. 1977–2016. DOI: [10.3982/ECTA9623](https://doi.org/10.3982/ECTA9623).
- [3] Alberto Alesina, Omar Barbiero, Carlo Favero, Francesco Giavazzi, and Matteo Paradisi. “The effects of fiscal consolidations: Theory and evidence.” In: (2017). DOI: [10.3386/w23385](https://doi.org/10.3386/w23385).
- [4] Alberto Alesina, Carlo Favero, and Francesco Giavazzi. *Austerity: When it Works and when it Doesn't*. Princeton University Press, 2020.
- [5] Alberto Alesina, Carlo Favero, and Francesco Giavazzi. “The output effect of fiscal consolidation plans.” In: *Journal of International Economics* 96 (2015), S19–S42. DOI: [10.1016/j.jinteco.2014.11.003](https://doi.org/10.1016/j.jinteco.2014.11.003).
- [6] Mario Alloza, Pablo Burriel, and Javier J Pérez. “Fiscal policies in the euro area: Revisiting the size of spillovers.” In: *Journal of Macroeconomics* 61 (2019), p. 103132.
- [7] Michele Aquaro, Natalia Bailey, and M Hashem Pesaran. “Estimation and inference for spatial models with heterogeneous coefficients: an application to US house prices.” In: *Journal of Applied Econometrics* 36.1 (2021), pp. 18–44.
- [8] Alan Auerbach, Yuriy Gorodnichenko, and Daniel Murphy. “Local fiscal multipliers and fiscal spillovers in the USA.” In: *IMF Economic Review* 68 (2020), pp. 195–229.
- [9] Guilherme Bandeira, Evi Pappa, Rana Sajedi, and Eugenia Vella. “Fiscal consolidation in a low-inflation environment: pay cuts versus lost jobs.” In: *International journal of central banking* 14 (2018), pp. 7–52.

- [10] David Rezza Baqaee and Emmanuel Farhi. “JEEA-FBBVA Lecture 2018: The Microeconomic Foundations of Aggregate Production Functions.” In: *Journal of the European Economic Association* 17.5 (2019), pp. 1337–1392.
- [11] David Rezza Baqaee and Emmanuel Farhi. *Macroeconomics with heterogeneous agents and input-output networks*. Tech. rep. National Bureau of Economic Research, 2018.
- [12] David Rezza Baqaee and Emmanuel Farhi. “The macroeconomic impact of microeconomic shocks: beyond Hulten’s Theorem.” In: *Econometrica* 87.4 (2019), pp. 1155–1203.
- [13] Alessandro Barattieri, Matteo Cacciatore, and Nora Traum. “Estimating the Effects of Government Spending Through the Production Network.” In: *Working Paper* (2023).
- [14] Jean-Noël Barrot and Julien Sauvagnat. “Input specificity and the propagation of idiosyncratic shocks in production networks.” In: *The Quarterly Journal of Economics* 131.3 (2016), pp. 1543–1592.
- [15] Roel Beetsma, Oana Furtuna, Massimo Giuliadori, and Haroon Mumtaz. “Revenue-versus spending-based fiscal consolidation announcements: Multipliers and follow-up.” In: *Journal of International Economics* 131 (2021), p. 103455.
- [16] Christoph E Boehm, Aaron Flaaen, and Nitya Pandalai-Nayar. “Input linkages and the transmission of shocks: Firm-level evidence from the 2011 Tōhoku earthquake.” In: *Review of Economics and Statistics* 101.1 (2019), pp. 60–75.
- [17] Benjamin Born, Gernot J Müller, and Johannes Pfeifer. “Does austerity pay off?” In: *Review of Economics and Statistics* 102.2 (2020), pp. 323–338.
- [18] Hafedh Bouakez, Omar Rachedi, and Emiliano Santoro. “The Government Spending Multiplier in a Multi-Sector Economy.” In: *American Economic Journal: Macroeconomics* (2023).
- [19] Edoardo Briganti and Victor Sellemi. “Who Anticipates Government Spending? Evidence from Defense Procurement.” In: *Working Paper* (2022).
- [20] Pedro Brinca, Miguel H Ferreira, Francesco Franco, Hans A Holter, and Laurence Malafry. “Fiscal consolidation programs and income inequality.” In: *International Economic Review* 62.1 (2021), pp. 405–460.

- [21] Matteo Cacciatore and Nora Traum. “Trade flows and fiscal multipliers.” In: *Review of Economics and Statistics* 104.6 (2022), pp. 1206–1223.
- [22] Yan Carrière-Swallow, Antonio C David, and Daniel Leigh. “Macroeconomic effects of fiscal consolidation in emerging economies: New Narrative evidence from Latin America and the Caribbean.” In: *Journal of Money, Credit and Banking* 53.6 (2021), pp. 1313–1335.
- [23] Vasco Carvalho. “Aggregate fluctuations and the network structure of intersectoral trade.” In: (2007).
- [24] Vasco Carvalho. “From micro to macro via production networks.” In: *Journal of Economic Perspectives* 28.4 (2014), pp. 23–48. DOI: [10.1257/jep.28.4.23](https://doi.org/10.1257/jep.28.4.23).
- [25] Vasco Carvalho and Alireza Tahbaz-Salehi. “Production networks: A primer.” In: *Annual Review of Economics* 11 (2019), pp. 635–663.
- [26] Juin-Jen Chang, Hsieh-Yu Lin, Nora Traum, and Shu-Chun S Yang. “Fiscal consolidation and public wages.” In: *Journal of Money, Credit and Banking* 53.2-3 (2021), pp. 503–533.
- [27] Giancarlo Corsetti, André Meier, and Gernot J Müller. “Cross-Border Spillovers from Fiscal Stimulus.” In: *International Journal of Central Banking* 6.1 (2010), pp. 5–37.
- [28] Lydia Cox, Gernot Muller, Ernesto Pasten, Raphael Schoenle, and Michael Weber. *Big g*. Tech. rep. National Bureau of Economic Research, 2020.
- [29] Era Dabla-Norris and Frederico Lima. “Macroeconomic effects of tax rate and base changes: Evidence from fiscal consolidations.” In: *European Economic Review* 153 (2023), p. 104399.
- [30] Michael B Devereux, Karine Gente, and Changhua Yu. “Production networks and international fiscal spillovers.” In: *The Economic Journal* 133.653 (2023), pp. 1871–1900.
- [31] Julian Di Giovanni and Galina Hale. “Stock market spillovers via the global production network: Transmission of US monetary policy.” In: *The Journal of Finance* 77.6 (2022), pp. 3373–3421.
- [32] George W Evans, Seppo Honkapohja, and Kaushik Mitra. “EXPECTATIONS, STAGNATION, AND FISCAL POLICY: A NONLINEAR ANALYSIS.” In: *International Economic Review* (2022).
- [33] Renato Faccini, Haroon Mumtaz, and Paolo Surico. “International fiscal spillovers.” In: *Journal of International Economics* 99 (2016), pp. 31–45.

- [34] Joel P Flynn, Christina Patterson, and John Sturm. *Fiscal policy in a networked economy*. Tech. rep. National Bureau of Economic Research, 2022.
- [35] Xavier Gabaix. “The granular origins of aggregate fluctuations.” In: *Econometrica* 79.3 (2011), pp. 733–772. DOI: [10.3982/ECTA8769](https://doi.org/10.3982/ECTA8769).
- [36] Ricardo Duque Gabriel, Mathias Klein, and Ana Sofia Pessoa. “The political costs of austerity.” In: *Review of Economics and Statistics* (2023), pp. 1–45.
- [37] Jaime Guajardo, Daniel Leigh, and Andrea Pescatori. “Expansionary austerity? International evidence.” In: *Journal of the European Economic Association* 12.4 (2014), pp. 949–968. DOI: [10.1111/jeea.12083](https://doi.org/10.1111/jeea.12083).
- [38] Galina Hale, Tumer Kapan, and Camelia Minoiu. “Shock transmission through cross-border bank lending: Credit and real effects.” In: (2019). DOI: [10.17016/FEDS.2019.052](https://doi.org/10.17016/FEDS.2019.052).
- [39] Karen J Horowitz, Mark A Planting, et al. *Concepts and Methods of the us input-Output Accounts*. Tech. rep. Bureau of Economic Analysis, 2006.
- [40] Ayobami E Ilori, Juan Paez-Farrell, and Christoph Thoenissen. “Fiscal policy shocks and international spillovers.” In: *European Economic Review* 141 (2022), p. 103969.
- [41] Omar Joya and Eric Rougier. “Do (all) sectoral shocks lead to aggregate volatility? Empirics from a production network perspective.” In: *European Economic Review* 113 (2019), pp. 77–107.
- [42] Madina Karamysheva. “How do fiscal adjustments work? An empirical investigation.” In: *Journal of Economic Dynamics and Control* 137 (2022), p. 104347.
- [43] Harry H Kelejian and Ingmar R Prucha. “A generalized moments estimator for the autoregressive parameter in a spatial model.” In: *International economic review* 40.2 (1999), pp. 509–533.
- [44] Mathias Klein and Ludger Linnemann. “Fiscal policy, international spillovers, and endogenous productivity.” In: (2021).
- [45] Lung-Fei Lee. “Asymptotic distributions of quasi-maximum likelihood estimators for spatial autoregressive models.” In: *Econometrica* 72.6 (2004), pp. 1899–1925.
- [46] Matthieu Lemoine and Jesper Lindé. “Fiscal consolidation under imperfect credibility.” In: *European Economic Review* 88 (2016), pp. 108–141.

- [47] James LeSage and Robert Pace. “Introduction to spatial econometrics.” In: (2009). DOI: [10.1111/j.1538-4632.2010.00797.x](https://doi.org/10.1111/j.1538-4632.2010.00797.x).
- [48] James LeSage and Olivier Parent. “Bayesian model averaging for spatial econometric models.” In: *Geographical Analysis* 39.3 (2007), pp. 241–267. DOI: [10.1111/j.1538-4632.2007.00703.x](https://doi.org/10.1111/j.1538-4632.2007.00703.x).
- [49] Karel Mertens and Morten Ravn. “Empirical Evidence on the Aggregate Effects of Anticipated and Unanticipated US Tax Policy Shocks.” In: *American Economic Journal: Economic Policy* 4.2 (2012), pp. 145–81.
- [50] Christopher J Nekarda and Valerie A Ramey. “Industry evidence on the effects of government spending.” In: *American Economic Journal: Macroeconomics* 3.1 (2011), pp. 36–59.
- [51] Keith Ord. “Estimation methods for models of spatial interaction.” In: *Journal of the American Statistical Association* 70.349 (1975), pp. 120–126. DOI: [10.1080/01621459.1975.10480272](https://doi.org/10.1080/01621459.1975.10480272).
- [52] Ali Ozdagli and Michael Weber. “Monetary policy through production networks: Evidence from the stock market.” In: (2017). DOI: [10.3386/w23424](https://doi.org/10.3386/w23424).
- [53] Evi Pappa, Rana Sajedi, and Eugenia Vella. “Fiscal consolidation with tax evasion and corruption.” In: *Journal of International Economics* 96 (2015), S56–S75.
- [54] Roberto Perotti. “Fiscal policy in good times and bad.” In: *The Quarterly Journal of Economics* 114.4 (1999), pp. 1399–1436.
- [55] Roberto Perotti. “In search of the transmission mechanism of fiscal policy.” In: *NBER Macroeconomics Annual, Chicago University Press* 22.1 (2007), pp. 169–226.
- [56] Andrea Pescatori, Mr Daniel Leigh, Jaime Guajardo, and Mr Pete Devries. “A new action-based dataset of fiscal consolidation.” In: 11-128 (2011). DOI: [10.5089/9781462346561.001.A001](https://doi.org/10.5089/9781462346561.001.A001).
- [57] Valerie A Ramey. “Ten years after the financial crisis: What have we learned from the renaissance in fiscal research?” In: *Journal of Economic Perspectives* 33.2 (2019), pp. 89–114.
- [58] Valerie A Ramey and Matthew D Shapiro. *Costly capital reallocation and the effects of government spending*. Tech. rep. National Bureau of Economic Research, 1999.

- [59] Christina D Romer and David H Romer. “The macroeconomic effects of tax changes: estimates based on a new measure of fiscal shocks.” In: *American Economic Review* 100.3 (2010), pp. 763–801. DOI: [10.1257/aer.100.3.763](https://doi.org/10.1257/aer.100.3.763).
- [60] Frank Smets, Joris Tielens, and Jan Van Hove. “Pipeline pressures and sectoral inflation dynamics.” In: (2019). DOI: [10.2139/ssrn.3346371](https://doi.org/10.2139/ssrn.3346371).
- [61] Erling Steigum and Øystein Thøgersen. “Borrow and adjust: Fiscal policy and sectoral adjustment in an open economy.” In: *International Economic Review* 44.2 (2003), pp. 699–724.
- [62] Quang H. Vuong. “Likelihood ratio tests for model selection and non-nested hypotheses.” In: *Econometrica* (1989). DOI: [10.2307/1912557](https://doi.org/10.2307/1912557).
- [63] Jeffrey M Wooldridge. *Econometric analysis of cross section and panel data*. MIT press, 2010.
- [64] Jihai Yu, Robert DeJong, and LungFei Lee. “Quasi-maximum likelihood estimators for spatial dynamic panel data with fixed effects when both n and T are large.” In: *Journal of Econometrics* 146.1 (2008), pp. 118–134.
- [65] Sarah Zubairy. “On fiscal multipliers: Estimates from a medium scale DSGE model.” In: *International Economic Review* 55.1 (2014), pp. 169–195.

Appendices

A Details on Aggregate Level Analysis

Impulse response functions are computed following the algorithm. Step 1 - solve dynamically forward the estimated equation putting all shocks to zero; step 2 - simulate the equation setting the fiscal adjustment plan to 1% of GDP; step 3 - compute impulse response as a difference between step 2 and step 1; step 4 - compute confidence intervals using the block bootstrap to take into account serial correlation.

In Section 2.2 we build impulse response functions using truncated moving average model. In particular we estimate the following specification:

$$\begin{aligned}\Delta y_t = & \alpha + B_1(L) \cdot f_t^u \cdot EB_t + B_2(L) f_t^u \cdot TB_t + \dots \\ & \dots + C_1(L) \cdot f_t^a \cdot EB_t + C_2(L) \cdot f_t^a \cdot TB_t + \dots \\ & \dots + \sum_{j=1}^H D_j \cdot f_{t,t+j}^a \cdot EB_t + \sum_{j=1}^H E_j \cdot f_{t,t+j}^a \cdot TB_t + \epsilon_t\end{aligned}$$

with:

$$\begin{aligned}f_{t,t+j}^a &= \delta_j^{TB} \cdot f_t^u \cdot TB_t + \epsilon_{t+j}^1, \text{ for } j = \overline{1, H} \\ f_{t,t+j}^a &= \delta_j^{EB} \cdot f_t^u \cdot EB_t + \epsilon_{t+j}^2, \text{ for } j = \overline{1, H}\end{aligned}$$

where $B(L)$ and $C(L)$ are polynomials of the length six, H - is the anticipation horizon and also equal to six. We follow Mertens and Ravn (2012) on this and six is the median implementation lag.

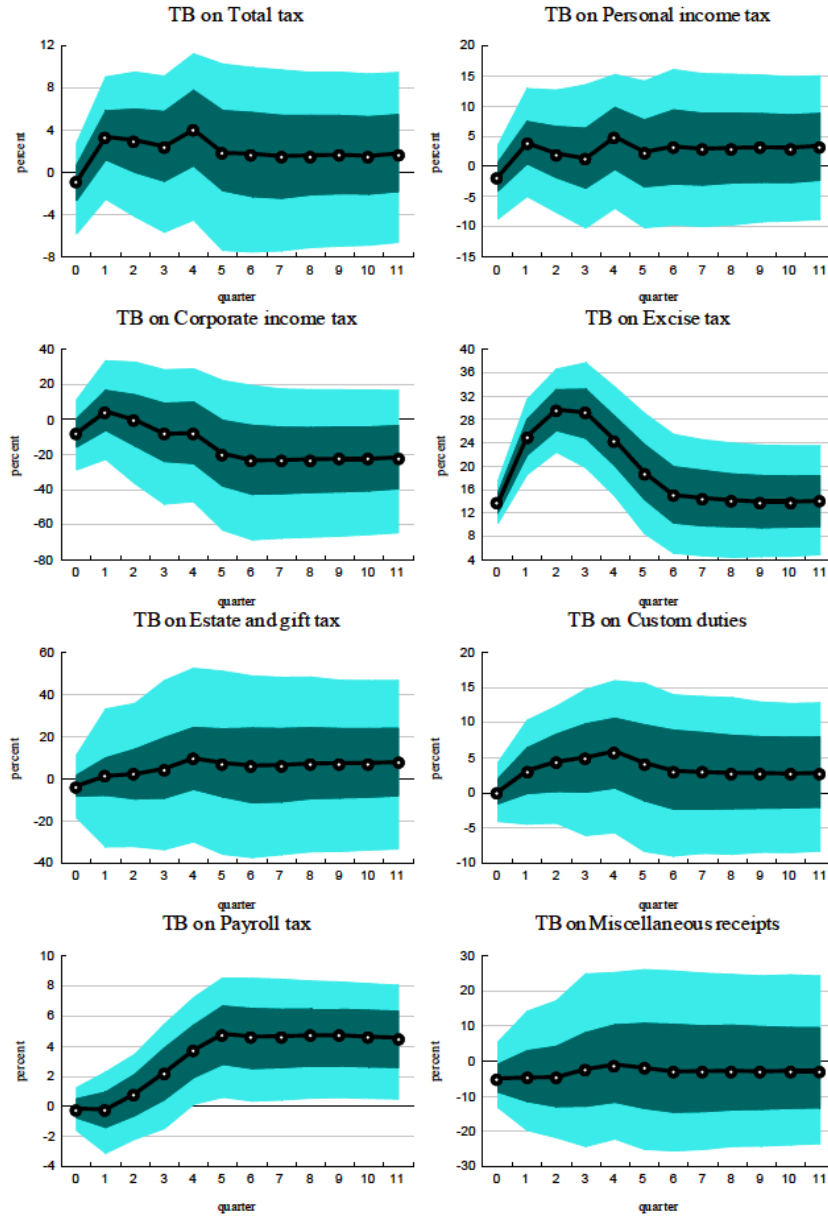
Figure 11 and 12 show the estimated impulse response functions of several other tax receipts shares of GDP to TB and EB fiscal adjustment plans. Those impulse responses are obtained using truncated moving average model.

B Industry Data

In this section we describe the data we use in our analysis.

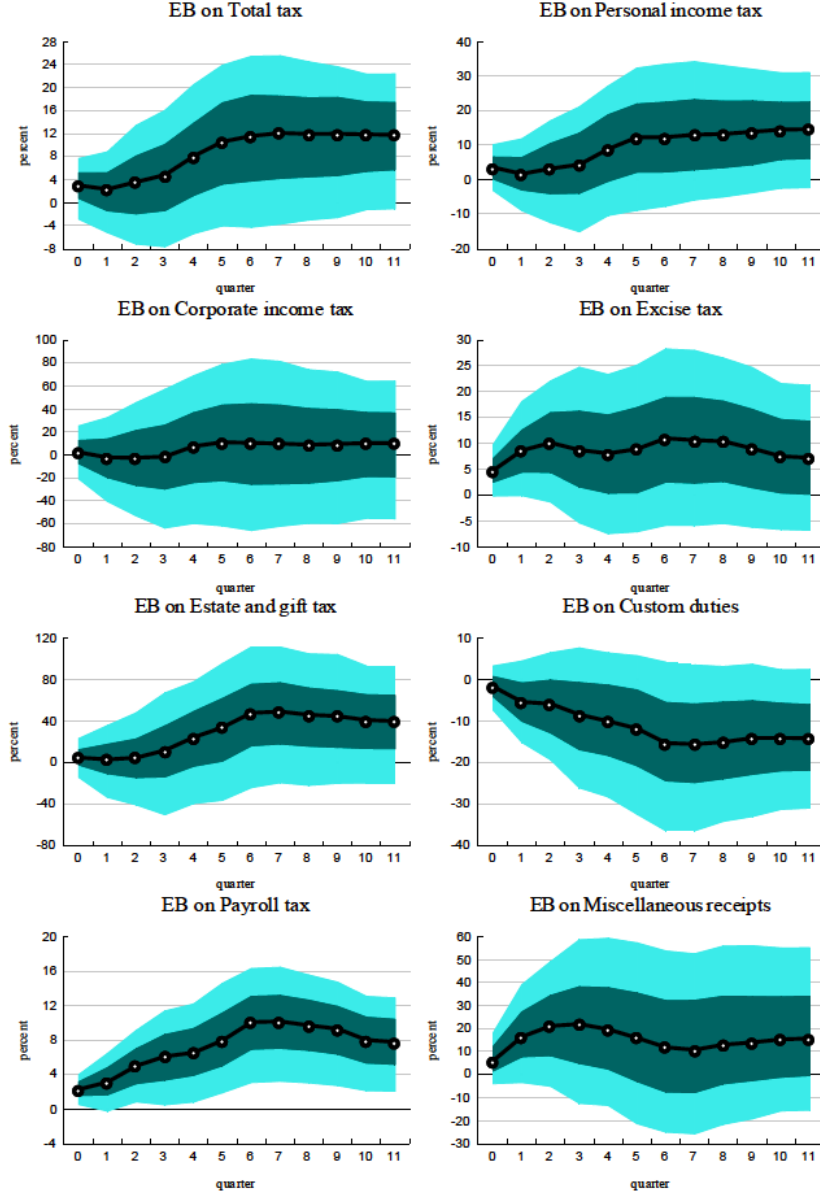
Firstly, the disaggregation level, $n = 62$, is determined by starting from the finest decomposition available on the Bureau of Economic Analysis (BEA) at a yearly frequency, namely 71 sectors, and then aggregating those sectors whose

Figure 11: Tax Receipts Response to TB Plans



data are not available for older years. We exclude the Government sector and consider only Government Enterprises as the only public, but politically independent, sector. The Government sector needs to be excluded since its outcome variable is G , government spending, which mechanically falls when a fiscal adjustment occurs.

Figure 12: Tax Receipts Response to EB Plans



Value Added

We use real industry value-added as the dependent variable, Δy_{it} . Value-added equals gross output minus intermediate inputs. It consists of compensation of employees, taxes on production and imports less subsidies (formerly indirect business taxes and non-tax payments), and gross operating surplus (formerly

“other value added”). We prefer it over gross output to be consistent with Acemoglu, Akcigit, and Kerr (2016).³⁰

Industry Specific Shares:

Following Acemoglu, Akcigit, and Kerr (2016), we construct the vector of industry-specific weights by exploiting information from the input-output tables, namely: $\omega_i^{EB} = \frac{Sales_{i \rightarrow G}}{Sales_i}$; where “G” stands for Government.³¹ By doing so, we take into account the fact that the government purchases goods and services in different quantities from each sector.³² Lastly, the vector of weights for the EB plan, denoted by ω^{EB} , is then normalized to one. On the contrary, we assume that aggregate TB fiscal plans impact each sector in the same fashion, therefore, we set $\omega_i^{TB} = 1/n$ for all i and the $n \times 1$ vector will be: $\omega^{TB} = 1/n \cdot \mathbf{1}_n$.

B.1 Input-Output Network

The BEA provides I-O tables that report the amount of commodity used (Use Table) and made (Make Table) by each industry. Horowitz, Planting, et al. (2006) outline the procedure to construct an industry-by-industry direct requirement matrix, with elements given by $SALES_{j \rightarrow i} / SALES_i$ for each sector. Let’s denote this matrix by A and note that its elements coincide one to one with the weights of $\Delta y_{i,t}^{\text{down}}$ in Equation 6. Therefore, the downstream spatial variable can be written in vector notation as: $\Delta \mathbf{y}_t^{\text{down}} = A \cdot \Delta \log \mathbf{y}_t$ and matrix A can be constructed from the Make and Use Tables of the BEA.³³ Henceforth we will refer to matrix A as the “downstream matrix”.

Finally, we construct a new matrix starting from A and using BEA’s industry specific gross output, such that its $(ij)_{th}$ element is represented by $SALES_{i \rightarrow j} / SALES_i$, which coincides one to one with the weights of $\Delta y_{i,t}^{\text{up}}$ in Equation 6. We denote this new matrix by \hat{A}^T , and refer to it as the “upstream matrix”. The upstream spatial variable can now be written in vector

³⁰Their decision is justified by the fact that value-added is adjusted for energy costs, non-manufacturing input, and inventory changes which are all outside of the general equilibrium model which provides the theoretical underpinning to their empirical strategy.

³¹Our definition of Government encompasses both Federal and State&Local government spending. We therefore exclude here Government Enterprises, which instead are considered as part of the industrial network.

³²We thank Roberto Perotti for this point.

³³We use the Make and Use tables of year 1997, which is the closest to the occurrence of fiscal plans. Nevertheless, notice that I-O matrices are fairly stable over time.

notation as: $\Delta \mathbf{y}_t^{\text{up}} = \hat{A}^T \cdot \Delta \log \mathbf{y}_t$.

The construction of matrices A and \hat{A}^T starts from the analysis of the Make and Use tables illustrated in chapter 12 of Horowitz, Planting, et al. (2006). We outline here the details of the construction and the precise mapping between the theory and the data.

The Use Table

The Use table is a commodity-by-industry table which illustrates the uses of commodities by intermediate and final users. The rows of the Use Table represent the commodities (or products) and the sum of the entries in a row is the total output of that commodity. On the contrary, the columns display the industries that employ them and the final users. Horowitz, Planting, et al. (2006) provides a useful numerical example with 3 industries:

Example of Use Table - 3 Industries					
Commodity/Industry	1	2	3	Final demand	Total Commodity Output
1	50	120	120	40	330
2	180	30	60	130	400
3	50	150	50	20	270
Scrap	1	3	1	0	5
VA	47	109	34	/	190
Total Industry Output	328	412	265	190	/

What is of our interest is clearly the $n \times n$ commodity-by-industry part of the Table, whose values can be denoted with the following notation:

$$(Use)_{ij} = \text{INP}_{i \rightarrow j} := \text{Commodity } i \text{ used as input by Industry } j$$

Therefore, the $n \times n$ part of the Use Table we are going to use is:

$$U = \begin{bmatrix} \text{INP}_{1 \rightarrow 1} & \text{INP}_{1 \rightarrow 2} & \text{INP}_{1 \rightarrow 3} \\ \text{INP}_{2 \rightarrow 1} & \text{INP}_{2 \rightarrow 2} & \text{INP}_{2 \rightarrow 3} \\ \text{INP}_{3 \rightarrow 1} & \text{INP}_{3 \rightarrow 2} & \text{INP}_{3 \rightarrow 3} \end{bmatrix} = \begin{bmatrix} 50 & 120 & 120 \\ 180 & 30 & 60 \\ 50 & 150 & 50 \end{bmatrix}$$

In practice, the above matrix U is a “symmetric” commodity-by-industry Use Table.

Next step boils down in constructing a *commodity-by-industry direct requirement table* by dividing each industry’s input, $\text{INP}_{j \rightarrow i}$, by its corresponding total industry output, y_i . We denote such a matrix with letter B :

$$B = \begin{bmatrix} \frac{\text{INP}_{1 \rightarrow 1}}{y_1} & \frac{\text{INP}_{1 \rightarrow 2}}{y_2} & \frac{\text{INP}_{1 \rightarrow 3}}{y_3} \\ \frac{\text{INP}_{2 \rightarrow 1}}{y_1} & \frac{\text{INP}_{2 \rightarrow 2}}{y_2} & \frac{\text{INP}_{2 \rightarrow 3}}{y_3} \\ \frac{\text{INP}_{3 \rightarrow 1}}{y_1} & \frac{\text{INP}_{3 \rightarrow 2}}{y_2} & \frac{\text{INP}_{3 \rightarrow 3}}{y_3} \end{bmatrix} = \begin{bmatrix} \frac{50}{328} & \frac{120}{412} & \frac{120}{265} \\ \frac{180}{328} & \frac{30}{412} & \frac{60}{265} \\ \frac{50}{328} & \frac{150}{412} & \frac{50}{265} \end{bmatrix} = \begin{bmatrix} 0.152 & 0.291 & 0.453 \\ 0.549 & 0.073 & 0.226 \\ 0.152 & 0.364 & 0.189 \end{bmatrix}.$$

Notice one important thing: matrix B is different from matrix A , since $x_{i \rightarrow j} \neq \text{INP}_{i \rightarrow j}$: the former is an industry output flow, while the second measures a commodity flow to an industry.

The Make Table

The Make table is an industry-by-commodity table which shows the production of commodities by industries. Row i represents an industry and its summation delivers the total industry output, y_i . Column j represents a commodity and its summation delivers the total commodity output.

Borrowing again Horowitz, Planting, et al., 2006’s 3 industries example, we have:

Example of Make Table - 3 Industries					
Industry/Commodity	1	2	3	Scrap	Total Industry Output
1	300	25	0	3	328
2	30	360	20	2	412
3	0	15	250	0	265
Total Commodity Output	330	400	270	5	/

Similarly to what done for the Use Table, we are interested in the central $n \times n$ elements of the table, which we can denote by V . The generic element of the “heart” of the Make table is:

$$(Make)_{ij} = \text{OUT}_{i \rightarrow j} := \text{Commodity } j \text{ produced by Industry } i$$

Therefore, the $n \times n$ part of the Make Table we are going to employ is:

$$V = \begin{bmatrix} \text{OUT}_{1 \rightarrow 1} & \text{OUT}_{1 \rightarrow 2} & \text{OUT}_{1 \rightarrow 3} \\ \text{OUT}_{2 \rightarrow 1} & \text{OUT}_{2 \rightarrow 2} & \text{OUT}_{2 \rightarrow 3} \\ \text{OUT}_{3 \rightarrow 1} & \text{OUT}_{3 \rightarrow 2} & \text{OUT}_{3 \rightarrow 3} \end{bmatrix} = \begin{bmatrix} 300 & 25 & 0 \\ 30 & 360 & 20 \\ 0 & 15 & 250 \end{bmatrix}$$

In practice, the above matrix V is a “symmetric” industry-by-commodity Make Table.

Analogously to what done before, we now take ratios; in particular, we divide each element of V by the total production of commodity j . The resulting matrix is denoted by D , and its generic element is:

$$(D)_{ij} = \frac{\text{OUT}_{i \rightarrow j}}{\sum_{k=1}^n \text{OUT}_{k \rightarrow j}} = \frac{\text{OUT}_{i \rightarrow j}}{C_j}$$

where $C_j := \sum_{k=1}^n \text{OUT}_{k \rightarrow j}$ is the total production of commodity j . D represents the share of industry i in the total production of commodity j ; not surprisingly, Horowitz, Planting, et al. (2006) refer to this matrix as the “market share matrix”. In the 3 industries/commodities example we have:

$$D = \begin{bmatrix} \frac{\text{OUT}_{1 \rightarrow 1}}{C_1} & \frac{\text{OUT}_{1 \rightarrow 2}}{C_2} & \frac{\text{OUT}_{1 \rightarrow 3}}{C_3} \\ \frac{\text{OUT}_{2 \rightarrow 1}}{C_1} & \frac{\text{OUT}_{2 \rightarrow 2}}{C_2} & \frac{\text{OUT}_{2 \rightarrow 3}}{C_3} \\ \frac{\text{OUT}_{3 \rightarrow 1}}{C_1} & \frac{\text{OUT}_{3 \rightarrow 2}}{C_2} & \frac{\text{OUT}_{3 \rightarrow 3}}{C_3} \end{bmatrix} = \begin{bmatrix} \frac{300}{330} & \frac{25}{400} & \frac{0}{270} \\ \frac{30}{330} & \frac{360}{400} & \frac{20}{270} \\ \frac{0}{330} & \frac{15}{400} & \frac{250}{270} \end{bmatrix} = \begin{bmatrix} 0.909 & 0.063 & 0 \\ 0.091 & 0.900 & 0.074 \\ 0 & 0.038 & 0.926 \end{bmatrix}$$

Adjustment for Scrap Products

The I-O accounts include a commodity for scrap, which is a byproduct of industry production. No industry produces scrap on demand; rather, it is the result of production to meet other demands. In order to make the I-O model work correctly, we have to eliminate scrap as a secondary product. At the same time, we must also keep industry output at the same level.

This adjustment is accomplished by calculating the ratio of non-scrap output to industry output for each industry and then applying these ratios to

the market shares matrix in order to account for total industry output. More precisely, the non-scrap ratio, which I denote by θ_i , is defined as follows:

$$\theta_i = \frac{y_i - (\text{scrap})_i}{y_i}$$

and represents the share of total industry output i made of commodity different from “scrap”. In the 3 industries example we have:

Industry	Tot.Ind.Out.	Scrap	Δ	θ_i
1	328	3	325	0.991
2	412	2	410	0.995
3	265	0	265	1

The market shares matrix, D , is adjusted for scrap by dividing each row by the non-scrap ratio for that industry. In the resulting transformation matrix, called W , the implicit commodity output of each industry has been increased. In other words, we are increasing each market share to take into account that to produce each unit of each commodity, industry i will produce $1/\theta_i$ units of output. In essence, we are spreading the production of commodity “scrap” over the production of all the other commodities:

$$W = \begin{bmatrix} \frac{OUT_{1 \rightarrow 1}}{C_1} \cdot \frac{1}{\theta_1} & \frac{OUT_{1 \rightarrow 2}}{C_2} \cdot \frac{1}{\theta_2} & \frac{OUT_{1 \rightarrow 3}}{C_3} \cdot \frac{1}{\theta_3} \\ \frac{OUT_{2 \rightarrow 1}}{C_1} \cdot \frac{1}{\theta_1} & \frac{OUT_{2 \rightarrow 2}}{C_2} \cdot \frac{1}{\theta_2} & \frac{OUT_{2 \rightarrow 3}}{C_3} \cdot \frac{1}{\theta_3} \\ \frac{OUT_{3 \rightarrow 1}}{C_1} \cdot \frac{1}{\theta_1} & \frac{OUT_{3 \rightarrow 2}}{C_2} \cdot \frac{1}{\theta_2} & \frac{OUT_{3 \rightarrow 3}}{C_3} \cdot \frac{1}{\theta_3} \end{bmatrix} = \begin{bmatrix} \frac{0.909}{0.991} & \frac{0.063}{0.991} & \frac{0}{0.991} \\ \frac{0.091}{0.995} & \frac{0.900}{0.995} & \frac{0.074}{0.995} \\ \frac{0}{1} & \frac{0.038}{1} & \frac{0.926}{1} \end{bmatrix} = \begin{bmatrix} 0.917 & 0.063 & 0 \\ 0.091 & 0.904 & 0 \\ 0 & 0.038 & 0.926 \end{bmatrix}$$

The Direct Requirement Table

To summarize:

1. We constructed matrix B , a commodity-by-industry direct requirement table, whose columns tell us how much an industry j needs of commodity i relative to its own total industry production.
2. We constructed matrix W , an industry-by-commodity matrix which represent the market share - adjusted for scrap - of each industry i in the production of a commodity j .

By combining these two matrices we can obtain an industry-by-industry direct requirement matrix:

$$\underbrace{P}_{\text{industry} \times \text{industry}} := \underbrace{W}_{\text{industry} \times \text{commodity}} \cdot \underbrace{B}_{\text{commodity} \times \text{industry}}$$

In order to understand the meaning of each element of matrix P, it is important to derive it analytically:

$$P = \underbrace{\begin{bmatrix} \frac{OUT_{1 \rightarrow 1}}{C_1 \cdot \theta_1} & \frac{OUT_{1 \rightarrow 2}}{C_2 \cdot \theta_2} & \frac{OUT_{1 \rightarrow 3}}{C_3 \cdot \theta_3} \\ \frac{OUT_{2 \rightarrow 1}}{C_1 \cdot \theta_1} & \frac{OUT_{2 \rightarrow 2}}{C_2 \cdot \theta_2} & \frac{OUT_{2 \rightarrow 3}}{C_3 \cdot \theta_3} \\ \frac{OUT_{3 \rightarrow 1}}{C_1 \cdot \theta_1} & \frac{OUT_{3 \rightarrow 2}}{C_2 \cdot \theta_2} & \frac{OUT_{3 \rightarrow 3}}{C_3 \cdot \theta_3} \end{bmatrix}}_W \cdot \underbrace{\begin{bmatrix} \frac{INP_{1 \rightarrow 1}}{y_1} & \frac{INP_{1 \rightarrow 2}}{y_2} & \frac{INP_{1 \rightarrow 3}}{y_3} \\ \frac{INP_{2 \rightarrow 1}}{y_1} & \frac{INP_{2 \rightarrow 2}}{y_2} & \frac{INP_{2 \rightarrow 3}}{y_3} \\ \frac{INP_{3 \rightarrow 1}}{y_1} & \frac{INP_{3 \rightarrow 2}}{y_2} & \frac{INP_{3 \rightarrow 3}}{y_3} \end{bmatrix}}_B$$

Denoting by p_{ij} the generic element of P, we have:

$$p_{ij} = \frac{\frac{OUT_{i \rightarrow 1}}{C_1 \cdot \theta_1} \cdot INP_{1 \rightarrow j} + \frac{OUT_{i \rightarrow 2}}{C_2 \cdot \theta_2} \cdot INP_{2 \rightarrow j} + \frac{OUT_{i \rightarrow 3}}{C_3 \cdot \theta_3} \cdot INP_{3 \rightarrow j}}{y_j} \approx \frac{SALES_{i \rightarrow j}}{SALES_j}$$

In other words, p_{ij} represents how much industry j depends on inputs from industry i relative to its own total industry output y_j .³⁴

Notice that the transposed of matrix P is approximately equal to matrix A in the paper:

$$P \approx \begin{bmatrix} \frac{SALES_{1 \rightarrow 1}}{SALES_1} & \frac{SALES_{1 \rightarrow 2}}{SALES_2} & \frac{SALES_{1 \rightarrow 3}}{SALES_3} \\ \frac{SALES_{2 \rightarrow 1}}{SALES_1} & \frac{SALES_{2 \rightarrow 2}}{SALES_2} & \frac{SALES_{2 \rightarrow 3}}{SALES_3} \\ \frac{SALES_{3 \rightarrow 1}}{SALES_1} & \frac{SALES_{3 \rightarrow 2}}{SALES_2} & \frac{SALES_{3 \rightarrow 3}}{SALES_3} \end{bmatrix} \Rightarrow \boxed{A \approx P^T}$$

³⁴Notice that a big assumption is made in the construction of this matrix: if industry i has adjusted market share of production of commodity K , $OUT_{i \rightarrow K} / (C_K \cdot \theta_K)$ equal to, say 10%, then it is assumed that if industry j purchases $z := INP_{K \rightarrow j}$ dollars of commodity K , then 10% of z \$ come from industry i . This must be true on average but it might not be exactly true case by case.

Matrix P can be either constructed from the Make and Use table or downloaded from the BEA, as an industry-by-industry direct requirement table. Its transposed value identifies the matrix A in Equation (6).

The construction of matrix \hat{A}^T , in equation (7), is trivial once we have matrix A as well as a vector of average industry output.

C Spatial Econometric Estimation

We believe that our empirical methodology presents some results of independent interest. Although we do not want to divert attention from the macroeconomic focus of the paper, we believe certain econometric facts are worth mentioning here in the Appendix. We provide this discussion in the spirit of promoting the usage of these new techniques in macroeconomic analysis.

Firstly, the adoption of spatial econometric methods allows us to disentangle the direct and network effect of aggregate shocks. This is a novel and recent innovation in macroeconomics, as noted in Ozdagli and Weber (2017). Secondly, spatial models are traditionally estimated by row-normalizing and removing the main diagonal from the weighting matrix. Another common assumption is homoskedasticity of the error term. In a recent paper, Aquaro, Bailey, and Pesaran (2021) develop a new estimator which relaxes homoskedasticity and allow for different spatial coefficients, thus indirectly relaxing the row-normalization assumption. They refer to it as Heterogenous Spatial Autoregressive model (HSAR). They also point out that not assuming zero entries on the main diagonal of the weighting matrix is simply a re-parameterization of the model, which does not harm the statistical properties of the MLE, but does change the interpretation of the parameters.³⁵ Their econometric model, adopted by Ozdagli and Weber (2017), is very convenient for macroeconomic applications which use non-row-normalized, dense main diagonal weighting matrices and in a setting where units are subject to heteroskedastic idiosyncratic shocks.

However, we highlight that even the standard dynamic spatial panel autoregressive model of J. Yu, DeJong, and Lee (2008) can easily be relaxed to accommodate for non-zero entries on the main diagonal and non-row-normalized weighting matrix with heteroskedastic errors.³⁶ Our construction of a Bayesian MCMC, similar to the one in LeSage and Pace (2009), is thus an easy and natural extension to the more general version of the spatial panel autoregressive

³⁵We are grateful to Hashem Pesaran for making us aware of this.

³⁶We thank Lung-Fei Lee for pointing this out.

model of J. Yu, DeJong, and Lee (2008). Moreover, the Bayesian MCMC method provides an easy way to recover the posterior distributions of the aggregate effects of the shocks, as illustrated earlier.

We encourage macroeconomists to adopt spatial econometric tools to study the propagation of aggregate shocks into a network of sub-units (countries, industries, regions...) but in doing so we also recommend them to follow three good practices:

1. Firstly, always allow for heteroskedasticity, since sub-units in general have different volatilities.
2. Secondly, never remove the main diagonal from the empirically observed weighting matrices, in our case A and \hat{A}^T . In fact, zero-entries in the main diagonal imposes a lack of spillovers within the same observed unit (“intra-unit feedback”). This is a reasonable assumption when units are individuals - like in standard spatial econometric applications - but it is not sensible when units are aggregates, such as industries. Notice, that the empirically observed A and \hat{A}^T weighting matrices from our analysis exhibit very dense main diagonals (see Figure 9).
3. Thirdly, never row-normalize the weighting matrices. Row-normalization flattens the differences in the degree of connection of each unit. For instance, in our application with the industrial network, A and \hat{A}^T exhibit very different row-sums, indicative of different degrees of exposure to customer and supplying industries.

We recommend using either the Bayesian MCMC methodology developed here and detailed in Appendix C.3 or the HSAR model of Aquaro, Bailey, and Pesaran (2021), whenever the application requires heterogeneous spatial coefficients. The relationship between the two models is left for future research.

In what follows we outline the details of the spatial econometric estimator that we employ.

C.1 Log-likelihood

The standard way to estimate the parameters of Equations (6) and (7) is via maximum likelihood (see LeSage and Pace (2009) for an introduction to spatial econometrics). The asymptotic and small sample properties of the MLE have been studied in Lee (2004) for cross-sectional data, and in J. Yu, DeJong, and Lee (2008), for dynamic panel data models with fixed effects.

We provide here the derivation of the log-likelihood of the baseline model (6), necessary for the calculation of both the MLE and the conditional posterior distributions of the Bayesian MCMC.³⁷ Collecting fiscal adjustment plans, industry fixed effects and other controls into matrix X_t , from Equation (6):

$$\begin{aligned} H_t^{-1} \cdot \Delta y_t &= \underset{n \times 1}{X_t} \cdot \underset{n \times k}{\beta} + \varepsilon_t \\ H_t &= \left(I_n - \rho^{down} \cdot A \cdot TB_t - \rho^{up} \cdot \hat{A}^T \cdot EB_t \right)^{-1} \\ \varepsilon_t &\sim \mathcal{N}(0, \Omega), \forall t \in \{1, \dots, T\} \\ \Omega &= \text{diag}(\sigma_1^2, \dots, \sigma_n^2) \\ \varepsilon_t &\perp \varepsilon_{t+i}, \quad \forall t \in \{1, \dots, T\}, \forall i \in \mathcal{Z} \end{aligned}$$

where k is the number of regressors.³⁸ We now make a convenient change in the notation: 1. we now use $'$ as a symbol for transposition instead of T ; 2. we now set $\rho_1 = \rho^{down}$, $\rho_2 = \rho^{up}$, $A = W_1$ and $\hat{A}' = W_2$. We have:

$$Z_t := H_t^{-1} \cdot \Delta y_t \sim \mathcal{N}(X_t \beta, \Omega) \implies \Delta y_t \sim \mathcal{N}(H_t X_t \beta, H_t \Omega H_t')$$

The density function of the random vector Δy_t is:

$$f(\Delta y_t | X_t, \rho, \beta, \Omega) = \frac{1}{\sqrt{(2\pi)^n \cdot |H_t \Omega H_t'|}} \exp \left\{ -\frac{1}{2} \cdot (\Delta y_t - H_t X_t \beta)' \cdot (H_t \Omega H_t')^{-1} \cdot (\Delta y_t - H_t X_t \beta) \right\},$$

with $\rho = [\rho^{down}, \rho^{up}]$.

Given that $(H_t \Omega H_t')^{-1} = (H_t')^{-1} \cdot \Omega^{-1} \cdot H_t^{-1}$ and $|H_t \Omega H_t'| = |H_t|^2 \cdot |\Omega|$, we have:

$$\begin{aligned} f(\Delta y_t | \cdot) &= (2\pi)^{-n/2} \cdot |H_t|^{-1} \cdot |\Omega|^{-1/2} \cdot \exp \left\{ -\frac{1}{2} (Z_t - X_t \beta)' \cdot H_t' \cdot (H_t')^{-1} \cdot \Omega^{-1} \cdot H_t^{-1} \cdot H_t \cdot (Z_t - X_t \beta) \right\} \\ &= (2\pi)^{-n/2} \cdot |(I_n - \rho_1 W_1 TB_t - \rho_2 W_2 EB_t)^{-1}|^{-1} \cdot |\Omega|^{-1/2} \exp \left\{ -\frac{1}{2} \varepsilon_t' \Omega^{-1} \varepsilon_t \right\} \\ &= (2\pi)^{-n/2} \cdot |I_n - \rho_1 \cdot W_1 \cdot TB_t - \rho_2 \cdot W_2 \cdot EB_t| \cdot |\Omega|^{-1/2} \exp \left\{ -\frac{1}{2} \varepsilon_t' \Omega^{-1} \varepsilon_t \right\}, \end{aligned}$$

At this point we need to find the likelihood of the random vector $\Delta y = [\Delta y_1' \dots \Delta y_T']$. Since the model is static and we have assumed $\text{cov}(\varepsilon_t, \varepsilon_{t-k}) =$

³⁷Results for the inverted model, Equation (7)) are symmetric to the baseline case.

³⁸ k in our baseline is n fixed effects plus 6 fiscal adjustment components (unexpected, announced and future for both TB and EB plans) plus 2 year dummies for 2008 and 2009.

$\rho^0_{n \times n}$, then Δy_t is *iid* over time. By consequence, the following holds:

$$f(\Delta y_{nT \times 1} | X_1, \dots, X_T, \rho, \beta, \Omega) = \prod_{t=1}^T f(\Delta y_t | X_t, \rho, \beta, \Omega) = ((2\pi)^n |\Omega|)^{-T/2} \cdot \prod_{t=1}^T |I_n - \rho_1 \cdot W_1 \cdot TB_t - \rho_2 \cdot W_2 \cdot EB_t| \exp \left\{ -\frac{1}{2} \cdot \sum_{t=1}^T \varepsilon_t' \Omega^{-1} \varepsilon_t \right\}.$$

Now we divide the time series of length T in three different sub-periods. In doing so, consider the following new parameters:

- t_1 : set of years when a tax based fiscal adjustment occurs.
Formally $t_1 := \{1, \dots, t, \dots, T_1 \mid t \text{ such that } TB_t = 1\}$. We set: $H_t \mid t \in t_1 = (I_n - \rho_1 \cdot W_1)^{-1} = H_\tau$.
- t_2 : set of years when an expenditure tax based fiscal adjustment occurs.
Formally: $t_2 := \{1, \dots, t, \dots, T_2 \mid t \text{ such that } EB_t = 1\}$. We set $H_t \mid t \in t_2 = (I_n - \rho_2 \cdot W_2)^{-1} = H_\gamma$.
- t_3 : set of years when neither a tax based fiscal adjustment nor an expenditure based fiscal adjustment occurs.
Formally $t_3 := \{1, \dots, t, \dots, T_3 \mid t \text{ such that } TB_t = 0 \wedge EB_t = 0\}$. We set $H_t \mid t \in t_3 = (I_n)^{-1} = I_n$.

Therefore, we have that t_1 , t_2 and t_3 account for a partition of the whole time series and $T = T_1 + T_2 + T_3$. By consequence we have:

$$\begin{aligned} \prod_{t=1}^T |I_n - \rho_1 W_1 TB_t - \rho_2 W_2 EB_t| &= \prod_{t=1}^T |H_t^{-1}| \\ &= \prod_{t=1}^T \frac{1}{|H_t|} \\ &= \prod_{t \in t_1}^{T_1} \frac{1}{|H_t|} \cdot \prod_{t \in t_2}^{T_2} \frac{1}{|H_t|} \cdot \prod_{t \in t_3}^{T_3} \frac{1}{|H_t|} \\ &= |H_\tau|^{-T_1} \cdot |H_\gamma|^{-T_2} \cdot |I_n|^{-T_3} \\ &= |I_n - \rho_1 \cdot W_1|^{T_1} \cdot |I_n - \rho_2 W_2|^{T_2} \end{aligned}$$

At this point, we rewrite the probability density function of our dependent

variable as:

$$f(\Delta y_t | X_1, \dots, X_T, \rho, \beta, \Omega) = (2\pi)^{-nT/2} \cdot |\Omega|^{-T/2} \cdot |I_n - \rho_1 \cdot W_1|^{T_1} \cdot |I_n - \rho_2 W_2|^{T_2} \cdot \exp \left\{ -\frac{1}{2} \cdot \sum_{t=1}^T \varepsilon'_t \cdot \Omega^{-1} \cdot \varepsilon_t \right\}.$$

Finally, the log-likelihood of our dataset is:

$$\begin{aligned} \log \mathcal{L}(\rho, \beta, \Omega | \Delta y_1, \dots, \Delta y_T, X_1, \dots, X_T) &= -\frac{nT}{2} \ln(2\pi) - \frac{T}{2} \cdot \ln(|\Omega|) + \\ &+ T_1 \cdot \ln(|I_n - \rho_1 \cdot W_1|) + T_2 \cdot \ln(|I_n - \rho_2 W_2|) - \frac{1}{2} \cdot \sum_{t=1}^T \varepsilon'_t \cdot \Omega^{-1} \cdot \varepsilon_t. \end{aligned}$$

with:

$$\varepsilon_t = Z_t - X_t \cdot \beta = H_t^{-1} \cdot \Delta y_t - X_t \beta = (I_n - \rho_1 W_1 T B_t - \rho_2 W_2 E B_t) \cdot \Delta y_t - X_t \cdot \beta.$$

Furthermore, we impose the condition $\lambda_{\min}^{-1} < \hat{\rho}_1 < \lambda_{\max}^{-1}$ and $\mu_{\min}^{-1} < \hat{\rho}_2 < \mu_{\max}^{-1}$, where λ and μ are the eigenvalues of the spatial matrices W_1 and W_2 respectively. This condition guarantees that the estimated model will have positive definite covariance matrix (see Ord (1975)).

Notice that in the inverted model of Equation (7), it is enough to switch the definition of W_1 and W_2 by setting: $A = W_2$ and $\hat{A}' = W_1$.

C.2 The Analytical Fisher Information Matrix

In order to derive the Fisher Information Matrix we firstly need to obtain the total gradient of the log-likelihood function. Let's start with the spatial coefficient ρ_1 :

$$\frac{\partial \log \mathcal{L}(\theta | \Delta y, X)}{\partial \rho_1} = T_1 \frac{1}{|I_n - \rho_1 W_1|} \frac{\partial |I_n - \rho_1 W_1|}{\partial \rho_1} - \frac{1}{2} \sum_{t=1}^T \frac{\partial (Z'_t \Omega^{-1} Z_t)}{\partial \rho_1} - 2 \frac{\partial (Z'_t \Omega^{-1} X_t \beta)}{\partial \rho_1}.$$

By some matrix algebra, it is possible to show that:

$$\begin{aligned} \frac{\partial (Z'_t \Omega^{-1} Z_t)}{\partial \rho_1} &= -T B_t \cdot \Delta y'_t \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t - T B_t \cdot \Delta y'_t \cdot W_1' \Omega^{-1} \cdot \Delta y_t \\ &+ 2\rho_1 \cdot T B_t^2 \cdot \Delta y'_t \cdot W_1 \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t + 2\rho_2 \cdot T B_t \cdot E B_t \cdot \Delta y'_t \cdot W_1 \cdot \Omega^{-1} \cdot W_2 \cdot \Delta y_t \end{aligned}$$

Since our fiscal adjustment plans are mutually exclusive, we have that $TB_t \cdot EB_t = 0$ for all t . Moreover, by rearranging the above expression, we get:

$$\frac{\partial(Z_t' \Omega^{-1} Z_t)}{\partial \rho_1} = -2 \cdot TB_t \cdot \Delta y_t' \cdot (I_n - \rho_1 \cdot W_1') \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t$$

After other matrix algebra, we get:

$$-2 \cdot \frac{\partial(Z_t \cdot \Omega^{-1} X_t \beta)}{\partial \rho_1} = 2 \cdot TB_t \cdot \Delta y_t' \cdot W_1' \cdot \Omega^{-1} \cdot X_t \cdot \beta$$

Wrapping up all together, and employing the notation introduced earlier: $(I_n - \rho_1 W_1)^{-1} = H_\tau$, we have:

$$\begin{aligned} \frac{\partial \log \mathcal{L}(\theta | \Delta y, X)}{\partial \rho_1} &= T_1 \frac{1}{|I_n - \rho_1 W_1|} \frac{\partial |I_n - \rho_1 W_1|}{\partial \rho_1} + \\ &+ \sum_{t \in t_1}^{T_1} \left[\Delta y_t' \cdot (I_n - \rho_1 \cdot W_1') \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t - \Delta y_t' \cdot W_1' \cdot \Omega^{-1} \cdot X_t \cdot \beta \right] = \\ &= T_1 \frac{1}{|I_n - \rho_1 W_1|} \cdot |I_n - \rho_1 W_1| \cdot Tr \left((I_n - \rho_1 W_1)^{-1} \cdot (-W_1) \right) + \\ &+ \sum_{t \in t_1}^{T_1} \left[((I_n - \rho_1 \cdot W_1) \cdot \Delta y_t)' \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t - \beta' \cdot X_t' \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t \right] \\ &= -T_1 \cdot Tr \left(H_\tau \cdot W_1 \right) + \sum_{t \in t_1}^{T_1} \left[(Z_t - X_t \beta)' \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t \right] \\ &= \sum_{t \in t_1}^{T_1} (\varepsilon_t' \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t) - T_1 \cdot Tr(H_\tau \cdot W_1). \end{aligned}$$

By simmetry we have that:

$$\frac{\partial \log \mathcal{L}(\theta | \Delta y, X)}{\partial \rho_2} = \sum_{t \in t_2}^{T_2} (\varepsilon_t' \cdot \Omega^{-1} \cdot W_2 \cdot \Delta y_t) - T_2 \cdot Tr(H_\gamma \cdot W_2),$$

with $H_\gamma = (I_n - \rho_2 W_2)^{-1}$, from the previous notation.

As far as concern the derivative with respect to β , we have already seen when

concentrating the log-likelihood that:

$$\begin{aligned}
\frac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \beta} &= X' \cdot \Sigma^{-1} \cdot Z - X' \cdot \Sigma^{-1} \cdot X \cdot \beta \\
&= X' \cdot \Sigma^{-1} \cdot (Z - X \cdot \beta) = \\
&= X' \cdot \Sigma^{-1} \cdot \varepsilon = \\
&= \sum_{t=1}^T X'_t \cdot \Omega^{-1} \cdot \varepsilon_t.
\end{aligned}$$

Concerning the derivatives with respect to σ_i^2 , we need firstly to acknowledge that:

$$\sum_{t=1}^T \varepsilon'_t \cdot \Omega^{-1} \cdot \varepsilon_t = \sum_{t=1}^T \sum_{i=1}^n \frac{\varepsilon_{i,t}^2}{\sigma_i^2} = \sum_{i=1}^n \frac{1}{\sigma_i^2} \sum_{t=1}^T \varepsilon_{i,t}^2,$$

and that:

$$\ln(|\Omega|) = \ln\left(\prod_{i=1}^n \sigma_i^2\right) = \sum_{i=1}^n \ln(\sigma_i^2).$$

Therefore, we have that:

$$\begin{aligned}
\frac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \sigma_i^2} &= -\frac{T}{2} \frac{\partial \ln(|\Omega|)}{\partial \sigma_i^2} - \frac{1}{2} \cdot \frac{\partial}{\partial \sigma_i^2} \sum_{t=1}^T \varepsilon'_t \cdot \Omega^{-1} \cdot \varepsilon_t \\
&= -\frac{T}{2 \cdot \sigma_i^2} + \frac{1}{2 \cdot \sigma_i^4} \cdot \sum_{t=1}^T \varepsilon_{i,t}^2.
\end{aligned}$$

We now have all the elements to write down the gradient of the log-likelihood:

$$\nabla \log \mathcal{L}(\theta|\Delta y, X) = \begin{bmatrix} \frac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_1} \\ \frac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_2} \\ \frac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \beta} \\ \frac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \sigma_1^2} \\ \vdots \\ \frac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \sigma_n^2} \end{bmatrix}_{38 \times 1} = \begin{bmatrix} \sum_{t \in t_1}^{T_1} (\varepsilon'_t \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t) - T_1 \cdot \text{Tr}(H_\tau \cdot W_1) \\ \sum_{t \in t_2}^{T_2} (\varepsilon'_t \cdot \Omega^{-1} \cdot W_2 \cdot \Delta y_t) - T_2 \cdot \text{Tr}(H_\gamma \cdot W_2) \\ \sum_{t=1}^T X'_t \cdot \Omega^{-1} \cdot \varepsilon_t \\ -\frac{T}{2 \cdot \sigma_1^2} + \frac{1}{2 \cdot \sigma_1^4} \cdot \sum_{t=1}^T \varepsilon_{1,t}^2 \\ \vdots \\ -\frac{T}{2 \cdot \sigma_n^2} + \frac{1}{2 \cdot \sigma_n^4} \cdot \sum_{t=1}^T \varepsilon_{n,t}^2 \end{bmatrix}$$

Another round of derivation is now needed. Let's start with the first row of the matrix: all the derivatives of $\frac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_1}$ with respect to all the parameters. To simplify notation we will refer with \mathcal{H}_{ij} to the element of row i and column j of the Hessian matrix.

$$\begin{aligned} \mathcal{H}_{1,1} &= \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_1^2} = \sum_{t \in t_1}^{T_1} \left(\frac{\partial \varepsilon'_t}{\partial \rho_1} \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t \right) - T_1 \cdot \frac{\partial \text{Tr}(H_\tau \cdot W_1)}{\partial \rho_1} \\ &= \sum_{t \in t_1}^{T_1} \left((-\Delta y'_t \cdot W'_1) \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t \right) - T_1 \cdot \text{Tr} \left(\frac{\partial H_\tau}{\partial \rho_1} \cdot W_1 \right) = \\ &= - \sum_{t \in t_1}^{T_1} \left(\Delta y'_t \cdot W'_1 \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t \right) - T_1 \cdot \text{Tr} \left((-H_\tau \cdot (-W_1) \cdot H_\tau) \cdot W_1 \right) = \\ &= -T_1 \cdot \text{Tr}(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau) - \sum_{t \in t_1}^{T_1} \left(\Delta y'_t \cdot W'_1 \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t \right) \end{aligned}$$

Symmetrically we have:

$$\begin{aligned}\mathcal{H}_{2,2} &= \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_2^2} = \\ &= -T_2 \cdot \text{Tr}(W_2 \cdot H_\gamma \cdot W_2 \cdot H_\gamma) - \sum_{t \in t_2}^{T_2} (\Delta y'_t \cdot W'_2 \cdot \Omega^{-1} \cdot W_2 \cdot \Delta y_t)\end{aligned}$$

Going back to the first row, we now calculate the cross derivative with respect to ρ_2 . Before doing so, recall that, being the log-likelihood a continuously differentiable function, the Schwarz's theorem applies and the Hessian matrix is symmetric.

$$\mathcal{H}_{1,2} = \mathcal{H}_{2,1} = \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_1 \partial \rho_2} = 0.$$

Going on with the calculation we have:

$$\begin{aligned}\mathcal{H}_{1,3:1,23} &= \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_1 \partial \beta} = \sum_{t \in t_1}^{T_1} \left(\frac{\partial \varepsilon'_t}{\partial \beta} \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t \right) \\ &= - \sum_{t \in t_1}^{T_1} X'_t \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t \\ &= -X'_\tau \cdot (I_{T_1} \otimes \Omega^{-1}) \cdot (I_{T_1} \otimes W_1) \cdot \Delta y_\tau\end{aligned}$$

Σ_τ^{-1}

where $\mathcal{H}_{1,3:1,23}$ means all the elements of the first row, from column 3 up to column 23. X_τ and Δy_τ represent X and Δy but for the only years when a tax based fiscal adjustment occur:

$$X_\tau = \begin{bmatrix} X_1 \\ \vdots \\ X_t \\ \vdots \\ X_{T_1} \end{bmatrix}_{T_1 n \times k} \quad \text{and} \quad \Delta y_\tau = \begin{bmatrix} \Delta y_1 \\ \vdots \\ \Delta y_t \\ \vdots \\ \Delta y_{T_1} \end{bmatrix}_{T_1 n \times k} \quad \text{with } t \in t_1,$$

Symmetrically:

$$\begin{aligned}
\mathcal{H}_{2,3:2,23} &= \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_2 \partial \beta} = \sum_{t \in t_2}^{T_2} \left(\frac{\partial \varepsilon'_t}{\partial \beta} \cdot \Omega^{-1} \cdot W_2 \cdot \Delta y_t \right) \\
&= - \sum_{t \in t_2}^{T_2} X'_t \cdot \Omega^{-1} \cdot W_2 \cdot \Delta y_t \\
&= -X'_\gamma \cdot (I_{T_2} \otimes \Omega^{-1}) \cdot (I_{T_2} \otimes W_2) \cdot \Delta y_\gamma,
\end{aligned}$$

Σ_γ^{-1}

with:

$$X_\gamma = \begin{bmatrix} X_1 \\ \vdots \\ X_t \\ \vdots \\ X_{T_2} \end{bmatrix}_{T_2 n \times k} \quad \text{and} \quad \Delta y_\gamma = \begin{bmatrix} \Delta y_1 \\ \vdots \\ \Delta y_t \\ \vdots \\ \Delta y_{T_2} \end{bmatrix}_{T_2 n \times k} \quad \text{with } t \in t_2,$$

$$\begin{aligned}
\mathcal{H}_{3,3:23,23} &= \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \beta^2} = \frac{\partial}{\partial \beta^2} \left(\sum_{t=1}^T X'_t \cdot \Omega^{-1} \cdot \varepsilon_t \right) \\
&= \sum_{t=1}^T X'_t \cdot \Omega^{-1} \cdot \frac{\partial (Z_t - X_t \cdot \beta)}{\partial \beta^2} \\
&= \sum_{t=1}^T X'_t \cdot \Omega^{-1} \cdot X_t \\
&= -X' \cdot \Sigma^{-1} \cdot X.
\end{aligned}$$

$$\mathcal{H}_{3,24:23,38} = \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \beta \partial \sigma^2} = \sum_{t=1}^T X'_t \cdot \frac{\partial \Omega^{-1}}{\partial \sigma^2} \cdot \varepsilon_t$$

The generic element of the above matrix is a $k \times 1$ vector:

$$-\sigma_1^{-4} \cdot \sum_{t=1}^T X'_{1,t} \cdot \varepsilon_{i,t}.$$

Going on with the calculation:

$$\mathcal{H}_{i,i|i \in [24,38]} = \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial(\sigma_i^2)^2} = \frac{T}{2} \cdot \frac{1}{\sigma_i^4} \cdot \left(1 - \frac{2}{T \cdot \sigma_i^2} \cdot \sum_{t=1}^T \varepsilon_{i,t}^2\right).$$

$$\mathcal{H}_{23+i,23+j|i,j \in [1,n]} = \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \sigma_i^2 \partial \sigma_j^2} = 0 \quad \forall i \neq j.$$

$$\begin{aligned} \mathcal{H}_{1,24:1,38} &= \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_1 \partial \sigma_i^2} = \frac{\partial}{\partial \sigma_i^2} \left(\sum_{t \in t_1}^{T_1} \varepsilon'_t \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t \right) \\ &= \frac{\partial}{\partial \sigma_i^2} \left(\sum_{t \in t_1}^{T_1} \text{Tr}(\varepsilon'_t \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t) \right) \\ &= \frac{\partial}{\partial \sigma_i^2} \left(\text{Tr} \left(\left(\sum_{t \in t_1}^{T_1} \Delta y_t \cdot \varepsilon'_t \right) \cdot \Omega^{-1} \cdot W_1 \right) \right) \\ &= \text{Tr} \left(\left(\sum_{t \in t_1}^{T_1} \Delta y_t \cdot \varepsilon'_t \right) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_i^2} \cdot W_1 \right) \end{aligned}$$

Note that

$$\frac{\partial \Omega^{-1}}{\partial \sigma_i^2} = \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & -\sigma_i^{-4} & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} = \text{diag}(0, \dots, 0, -\sigma_i^{-4}, 0, \dots, 0)$$

Symmetrically:

$$\mathcal{H}_{2,24:2,38} = \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_2 \partial \sigma_i^2} = \text{Tr} \left(\left(\sum_{t \in t_2}^{T_2} \Delta y_t \cdot \varepsilon'_t \right) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_i^2} \cdot W_2 \right)$$

At this point we have all the elements to construct the Hessian matrix of the log-likelihood.

To sum up, first row:

- $\mathcal{H}_{1,1} = -T_1 \cdot \text{Tr}(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau) - \sum_{t \in t_1}^{T_1} (\Delta y'_t \cdot W'_1 \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t)$
- $\mathcal{H}_{1,2} = 0$

- $\mathcal{H}_{1,3:1,23} = -\sum_{t \in t_1}^{T_1} X'_t \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t$
- $\mathcal{H}_{1,24:1,38} = Tr\left(\left(\sum_{t \in t_1}^{T_1} \Delta y_t \cdot \varepsilon'_t\right) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_i^2} \cdot W_1\right).$

Second row:

- $\mathcal{H}_{2,1} = 0$
- $\mathcal{H}_{2,2} = -T_2 \cdot Tr(W_2 \cdot H_\gamma \cdot W_2 \cdot H_\gamma) - \sum_{t \in t_2}^{T_2} (\Delta y'_t \cdot W'_2 \cdot \Omega^{-1} \cdot W_2 \cdot \Delta y_t)$
- $\mathcal{H}_{2,3:2,23} = -\sum_{t \in t_2}^{T_2} X'_t \cdot \Omega^{-1} \cdot W_2 \cdot \Delta y_t$
- $\mathcal{H}_{2,24:2,38} = Tr\left(\left(\sum_{t \in t_2}^{T_2} \Delta y_t \cdot \varepsilon'_t\right) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_i^2} \cdot W_2\right).$

From row 3 to row 23:

- $\mathcal{H}_{3,1:23,1} = \mathcal{H}'_{1,3:1,23}$
- $\mathcal{H}_{3,2:23,2} = \mathcal{H}'_{2,3:2,23}$
- $\mathcal{H}_{3,3:23,23} = \sum_{t=1}^T X'_t \cdot \Omega^{-1} \cdot X_t$
- $\mathcal{H}_{3,24:23,38} = \sum_{t=1}^T X'_t \cdot \frac{\partial \Omega^{-1}}{\partial \sigma^2} \cdot \varepsilon_t$

From row 24 to the last row (number 38):

- $\mathcal{H}_{24,1:38,1} = \mathcal{H}'_{1,24:1,38}$
- $\mathcal{H}_{24,2:38,2} = \mathcal{H}'_{2,24:2,38}$
- $\mathcal{H}_{24,3:38,23} = \mathcal{H}'_{3,24:23,38}$

$$\bullet \mathcal{H}_{23+i,23+j|i,j \in [1,n]} = \begin{cases} \frac{T}{2} \cdot \frac{1}{\sigma_i^4} \cdot \left(1 - \frac{2}{T \cdot \sigma_i^2} \cdot \sum_{t=1}^T \varepsilon_{i,t}^2\right) & \forall i = j \in [1, n] \\ 0 & \forall i \neq j \end{cases}$$

The last step we have to make to finally obtain the Fisher Information Matrix is taking expectations of every element.

$$\begin{aligned} E[\mathcal{H}_{1,1}] &= -T_1 \cdot \text{Tr}(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau) - \sum_{t \in t_1}^{T_1} E[\Delta y'_t \cdot W'_1 \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t] = \\ &= -T_1 \cdot \text{Tr}(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau) - \sum_{t \in t_1}^{T_1} E[\text{Tr}(W_1 \cdot \Delta y_t \cdot \Delta y'_t \cdot W'_1 \cdot \Omega^{-1})] = \\ &= -T_1 \cdot \text{Tr}(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau) - \sum_{t \in t_1}^{T_1} \text{Tr}(W_1 \cdot E[\Delta y_t \cdot \Delta y'_t] \cdot W'_1 \cdot \Omega^{-1}) = \\ &= -T_1 \cdot \text{Tr}(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau) - \sum_{t \in t_1}^{T_1} \text{Tr}(W_1 \cdot E[H_\tau \cdot X_t \cdot \beta \cdot \varepsilon'_t \cdot H'_\tau + \\ &\quad + H_\tau \cdot X_t \cdot \beta \cdot \beta' \cdot X'_t \cdot H'_\tau + H_\tau \cdot \varepsilon_t \cdot \varepsilon'_t \cdot H'_\tau \cdot \varepsilon_t \cdot \beta' \cdot X'_t \cdot H'_\tau] \cdot W'_1 \cdot \Omega^{-1}) = \\ &= -T_1 \cdot \text{Tr}(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau) - \sum_{t \in t_1}^{T_1} \text{Tr}(W_1 \cdot [H_\tau \cdot X_t \cdot \beta \cdot E[\varepsilon'_t] \cdot H'_\tau + \\ &\quad + H_\tau \cdot X_t \cdot \beta \cdot \beta' \cdot X'_t \cdot H'_\tau + H_\tau \cdot E[\varepsilon_t \cdot \varepsilon'_t] \cdot H'_\tau + E[\varepsilon_t] \cdot \beta' \cdot X'_t \cdot H'_\tau] \cdot W'_1 \cdot \Omega^{-1}) = \\ &= -T_1 \cdot \text{Tr}(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau) - \\ &\quad - \sum_{t \in t_1}^{T_1} \text{Tr}(W_1 \cdot [H_\tau \cdot X_t \cdot \beta \cdot \beta' \cdot X'_t \cdot H'_\tau + H_\tau \cdot \Omega \cdot H'_\tau] \cdot W'_1 \cdot \Omega^{-1}) = \\ &= -T_1 \cdot \text{Tr}(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau) - \\ &\quad - \sum_{t \in t_1}^{T_1} \text{Tr}(W_1 \cdot H_\tau \cdot X_t \cdot \beta \cdot \beta' \cdot X'_t \cdot H'_\tau \cdot W'_1 \cdot \Omega^{-1} + W_1 \cdot H_\tau \cdot \Omega \cdot H'_\tau \cdot W'_1 \cdot \Omega^{-1}) = \\ &= -T_1 \cdot \text{Tr}(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau + H'_\tau \cdot W'_1 \cdot \Omega^{-1} \cdot W_1 \cdot H_\tau \cdot \Omega) - \\ &\quad - \sum_{t \in t_1}^{T_1} \text{Tr}(\beta' \cdot X'_t \cdot H'_\tau \cdot W'_1 \cdot \Omega^{-1} \cdot W_1 \cdot H_\tau \cdot X_t \cdot \beta) = \end{aligned}$$

Setting $M_1^\tau = H'_\tau \cdot W'_1 \cdot \Omega^{-1} \cdot W_1 \cdot H_\tau$ we can rewrite the above identity as:

$$\begin{aligned} E[\mathcal{H}_{1,1}] &= -T_1 \cdot \text{Tr}(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau + M_1^\tau \cdot \Omega) - \sum_{t \in t_1}^{T_1} \beta' \cdot X'_t \cdot M_1^\tau \cdot X_t \cdot \beta = \\ &= -T_1 \cdot \text{Tr}(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau + M_1^\tau \cdot \Omega) - \beta' \cdot X'_\tau \cdot (I_{T_1} \otimes M_1^\tau) \cdot X_\tau \cdot \beta. \end{aligned}$$

Simmetrically:

$$E[\mathcal{H}_{2,2}] = -T_2 \cdot \text{Tr}(W_2 \cdot H_\gamma \cdot W_2 \cdot H_\gamma + M_1^\gamma \cdot \Omega) - \beta' \cdot X'_\gamma \cdot (I_{T_2} \otimes M_1^\gamma) \cdot X_\gamma \cdot \beta.$$

with $M_1^\gamma = H'_\gamma \cdot W'_2 \cdot \Omega^{-1} \cdot W_2 \cdot H_\gamma$.

Going on with the calculation:

$$\begin{aligned} E[\mathcal{H}_{1,3:1,23}] &= E\left[-\sum_{t \in t_1}^{T_1} X'_t \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t\right] = \\ &= -\sum_{t \in t_1}^{T_1} X'_t \cdot \Omega^{-1} \cdot W_1 \cdot E\left[H_\tau \cdot X_t \cdot \beta + H_\tau \cdot \varepsilon_t\right] = \\ &= -\sum_{t \in t_1}^{T_1} X'_t \cdot \Omega^{-1} \cdot W_1 \cdot H_\tau \cdot X_t \cdot \beta \\ &= X'_\tau \cdot (I_{T_1} \otimes M_2^\tau) \cdot X_\tau \cdot \beta \end{aligned}$$

with $M_2^\tau = \Omega^{-1} \cdot W_1 \cdot H_\tau$.

Simmetrically:

$$E[\mathcal{H}_{2,3:2,23}] = X'_\gamma \cdot (I_{T_2} \otimes M_2^\gamma) \cdot X_\gamma \cdot \beta$$

with $M_2^\gamma = \Omega^{-1} \cdot W_2 \cdot H_\gamma$.

Next step:

$$\begin{aligned}
E[\mathcal{H}_{1,24:1,38}] &= Tr\left(\left(\sum_{t \in t_1}^{T_1} E[\Delta y_t \cdot \varepsilon'_t]\right) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_i^2} \cdot W_1\right) = \\
&= Tr\left(\left(\sum_{t \in t_1}^{T_1} E[\Delta y_t \cdot \varepsilon'_t]\right) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_i^2} \cdot W_1\right) = \\
&= Tr\left(\left(\sum_{t \in t_1}^{T_1} H_\tau \cdot E[\varepsilon_t \cdot \varepsilon'_t]\right) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_i^2} \cdot W_1\right) = \\
&= T_1 \cdot Tr\left(H_\tau \cdot \Omega \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_i^2} \cdot W_1\right) = \\
&= T_1 \cdot Tr\left(\Omega \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_i^2} \cdot W_1 \cdot H_\tau\right),
\end{aligned}$$

Notice that

$$\Omega \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_i^2} = -\sigma_i^2 \cdot I_{ii}$$

where the generic element of matrix I_{ii} is given by

$$\omega_{s,t} = \begin{cases} 1 & s = i, j = i \\ 0 & \text{otherwise} \end{cases}$$

Therefore

$$\begin{aligned}
E[\mathcal{H}_{1,23+i}] &= T_1 \cdot \sigma_i^{-2} \cdot Tr\left(I_{ii} \cdot W_1 \cdot H_\tau\right) = \\
&= T_1 \cdot \sigma_i^{-2} \cdot \left(W_1 \cdot H_\tau\right)_{ii}
\end{aligned}$$

Finally we have that:

$$E[\mathcal{H}_{1,24:1:38}] = T_1 \cdot diag\left(\Omega^{-1} \cdot W_1 \cdot H_\tau\right) = T_1 \cdot diag(M_2^\tau).$$

Simmetrically:

$$E[\mathcal{H}_{2,24:2:38}] = T_2 \cdot diag\left(\Omega^{-1} \cdot W_2 \cdot H_\gamma\right) = T_2 \cdot diag(M_2^\gamma).$$

Going on:

$$\begin{aligned}
E[\mathcal{H}_{3,3:23,23}] &= E\left[\sum_{t=1}^T X'_t \cdot \Omega^{-1} \cdot X_t\right] = \sum_{t=1}^T X'_t \cdot \Omega^{-1} \cdot X_t = X' \cdot \Sigma^{-1} \cdot X \\
E[\mathcal{H}_{3,24:23,38}] &= E\left[\sum_{t=1}^T X'_t \cdot \frac{\partial \Omega^{-1}}{\partial \sigma^2} \cdot \varepsilon_t\right] \\
&= \sum_{t=1}^T X'_t \cdot \frac{\partial \Omega^{-1}}{\partial \sigma^2} \cdot E[\varepsilon_t] \\
&= \mathbf{0}_{k \times n} \\
E[\mathcal{H}_{23+i,23+j|i,j \in [1,n]}] &= \begin{cases} \frac{T}{2} \cdot \frac{1}{\sigma_i^4} \cdot \left(1 - \frac{2}{T \cdot \sigma_i^2} \cdot \sum_{t=1}^T E[\varepsilon_{i,t}^2]\right) & \forall i = j \in [1, n] \\ 0 & \forall i \neq j \end{cases} \\
&= \begin{cases} -\frac{T}{2} \cdot \frac{1}{\sigma_i^4} & \forall i = j \in [1, n] \\ 0 & \forall i \neq j \end{cases} \\
&= -\frac{T}{2} \cdot \begin{bmatrix} \sigma_1^{-4} & 0 & \dots & 0 \\ 0 & \sigma_2^{-4} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_n^{-4} \end{bmatrix} = -\frac{T}{2} \cdot V
\end{aligned}$$

We finally have all the elements of the Fisher Information Matrix for our panel (with dummy variables) spatial model:

$$\mathcal{J} =$$

$$\begin{bmatrix}
-T_1 \cdot Tr(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau + M_1^T \cdot \Omega) - \\
-\beta' \cdot X'_\tau \cdot (I_{T_1} \otimes M_1^T) \cdot X_\tau \cdot \beta \\
0 \\
-T_2 \cdot Tr(W_2 \cdot H_\gamma \cdot W_2 \cdot H_\gamma + M_1^T \cdot \Omega) - \\
-\beta' \cdot X'_\gamma \cdot (I_{T_2} \otimes M_1^T) \cdot X_\gamma \cdot \beta \\
X'_\tau \cdot (I_{T_1} \otimes M_2^T) \cdot X_\tau \cdot \beta \\
T_1 \cdot diag(M_2^T) \\
(X'_\tau \cdot (I_{T_1} \otimes M_2^T) \cdot X_\tau \cdot \beta)' \\
0 \\
(X'_\gamma \cdot (I_{T_2} \otimes M_2^T) \cdot X_\gamma \cdot \beta)' \\
X' \cdot \Sigma^{-1} \cdot X \\
T_2 \cdot diag(M_2^T) \\
\mathbf{0}_{n \times k} \\
T_1 \cdot diag(M_2^T) \\
T_2 \cdot diag(M_2^T) \\
\mathbf{0}_{k \times n} \\
-\frac{T}{2} \cdot V \\
T_1 \cdot diag(M_2^T) \\
T_2 \cdot diag(M_2^T) \\
\mathbf{0}_{k \times n} \\
-\frac{T}{2} \cdot V
\end{bmatrix}$$

C.3 Bayesian MCMC - Technical Details

Even if the MLE is a common standard method in spatial econometric applications, we have two valid reasons for not adopting it: 1. non-stationary estimates of aggregate total effects; 2. prior information on the values of the parameters. Let's explore both the issues.

1. Non-Stationary Solutions

We can estimate the parameters by maximizing the concentrated log-likelihood over the compact set which guarantees a positive definite matrix (see Ord (1975)): $C^{down} = (\lambda_{\min}^{-1}, \lambda_{\max}^{-1})$ and $C^{up} = (\mu_{\min}^{-1}, \mu_{\max}^{-1})$. The standard errors are constructed using the analytical Fisher Information of the model, centered on the point estimates, $\hat{\rho}^{down}$ and $\hat{\rho}^{up}$. The asymptotic results of J. Yu, De-Jong, and Lee (2008) guarantees the asymptotic normality of the parameters of equation (6) and (7) (See Theorem 3 case $n/T \rightarrow 0$). For instance, for the estimator of ρ^{down} we have:

$$\sqrt{T \cdot n} (\hat{\rho}_{nT}^{down} - \rho^{down}) \xrightarrow{d} \mathcal{N}(0, \sigma^2)$$

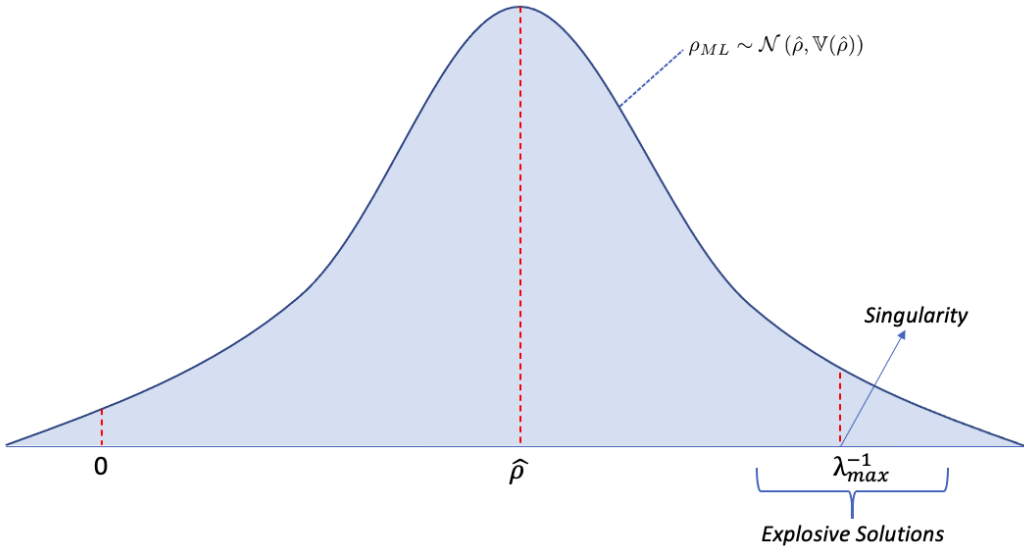
where σ^2 is the asymptotic variance of the MLE, obtained by the calculating the analytical Fisher Information matrix of our model. However, we are interested in estimating the aggregate total effect of fiscal consolidations, not the parameters of the model themselves. At page 70, LeSage and Pace (2009) suggest to construct the asymptotic distribution of the average total effect (our aggregate total effect) by following these steps: 1. estimate the parameters of the model via MLE; 2. Draw values of the parameters by their approximate asymptotic distribution ($\tilde{\rho}_{nT}^{down} \approx \mathcal{N}(\hat{\rho}_{nT}^{down}, \frac{\hat{\sigma}^2(\hat{\rho}_{nT}^{down})}{nT})$); 3. Calculate at each step the aggregate total effect. After doing so we calculated the standard errors of the ATE_{TB} by calculating the standard deviation of the asymptotic distribution so constructed. We obtained explosive solutions. This is a surprising result, in fact, the asymptotic normality of the average effect is guaranteed by the Δ -method:

$$\sqrt{T \cdot n} (ATE_{TB}(\hat{\rho}_{nT}^{down}) - ATE_{TB}(\rho^{down})) \xrightarrow{d} \mathcal{N}\left(0, \sigma^2 \cdot \left(\frac{\partial ATE_{TB}(\rho^{down})}{\partial \rho^{down}}\right)^2\right)$$

where $ATE_{TB} : C^{down} \rightarrow \mathbb{R}$ and $ATE_{TB}(x) = v' \cdot (I_n - x \cdot A)^{-1} \cdot \omega_{TB}$ and v is a vector of industry output shares of total industrial production (the weights we use to calculate the aggregate effect of fiscal consolidations). What goes wrong in this procedure? The Δ -method is an asymptotic result, which

might provide a terrible approximation of a finite sample distribution. It all boils down in finding a distribution which approximates well the small sample one. If $\hat{\rho}_{nT}^{down}$ is very closed to the boundary and its asymptotically normal standard errors are large, that is, they approach the boundary of C^{down} then we end up drawing values of ρ^{down} which deliver unrealistically large values of ATE_{TB} , because matrix $(I_n - \rho^{down} \cdot A)^{-1}$ becomes singular (the boundary is one eigenvalue of A). This situation is described in Figure 13.

Figure 13: Explosive Solutions of ATE_{TB}



2. Prior Information

We have two extra “prior” pieces of information on the value of the spatial parameters, ρ^{down} and ρ^{up} :

- i. Values of ρ^{down} and ρ^{up} close to the boundaries will deliver unrealistically high values of ATE, ADE and ANE, since the determinant of matrices $(I_n - \rho^{down} \cdot A)$ and $(I_n - \rho^{up} \cdot \hat{A}^T)$ will approach zero by definition of eigenvalue. In turn, the elements of their inverse matrices will explode, as illustrated above. Therefore, we should assign less weight to values of ρ^{down} and ρ^{up} close to the boundaries.
- ii. We know that industries that are close to each other in the production network will co-move. For instance, if industry X faces increasing prices

for its input, it will shrink production and increase prices; in turn, customers of X will also face the same problem and will react similarly, by reducing production and increasing prices. Therefore, the direction of the spatial correlation among industries' output is positive: $\rho^{down} > 0$ and $\rho^{up} > 0$.

Model Estimation

We can integrate such prior information into our estimation and avoid non-stationarity aggregate effects, by adopting a Bayesian MCMC similar to the one introduced by LeSage and Parent (2007). We illustrate here how we implement the Bayesian MCMC to estimate the parameters of Equation (6) (baseline). The log-likelihood of that model is the one outlined above. The priors we employ on the parameters are:

$$\begin{aligned}\pi(\beta) &\propto \text{constant} \\ \Omega &= \sigma^2 \cdot V \quad \text{with } V = \text{diag}(v_1, \dots, v_n) \\ \pi(\sigma^2) &\propto \frac{1}{\sigma^2} \\ \pi(v_i) &\stackrel{iid}{\sim} \Gamma^{-1}\left(\frac{r}{2}, \frac{r}{2}\right), \quad i = 1, \dots, n \\ \rho^{down} &\sim \text{Gen.Beta}(d, d) \\ \rho^{up} &\sim \text{Gen.Beta}(d, d).\end{aligned}$$

We adopt non-informative priors for σ^2 and β to reflect our lack of information around the values of these parameters. Concerning r , a lower value generates more diffusion in the distributions of v_i , thus regulating our confidence towards heteroskedasticity. Unlike LeSage and Pace (2009), who suggest a value of 4, we set r equal to 3 to reflect a strong belief towards heteroskedasticity. For instance, industries in the Agriculture (NAICS 11) as well as Mining (NAICS 21) macro sectors, exhibit much higher volatilities than the rest of the industries.

We impose a “generalized (or non-standardized) $\text{Beta}(d, d)$ prior”, with support from 0 to λ_{max}^{-1} for ρ^{down} and from 0 to $\hat{\lambda}_{max}^{-1}$ for ρ^{up} . We follow LeSage and Pace (2009) and set d equal to 1.1; which has the benefit of letting the generalized Beta prior to resemble a Uniform distribution (diffuse prior), but with low density at the boundaries, as illustrated in Figure 14. The choice of such a prior allows us to be agnostic about the specific value of the spatial parameters but at the same time it allows to embed the prior information we

Figure 14: Generalized Beta prior

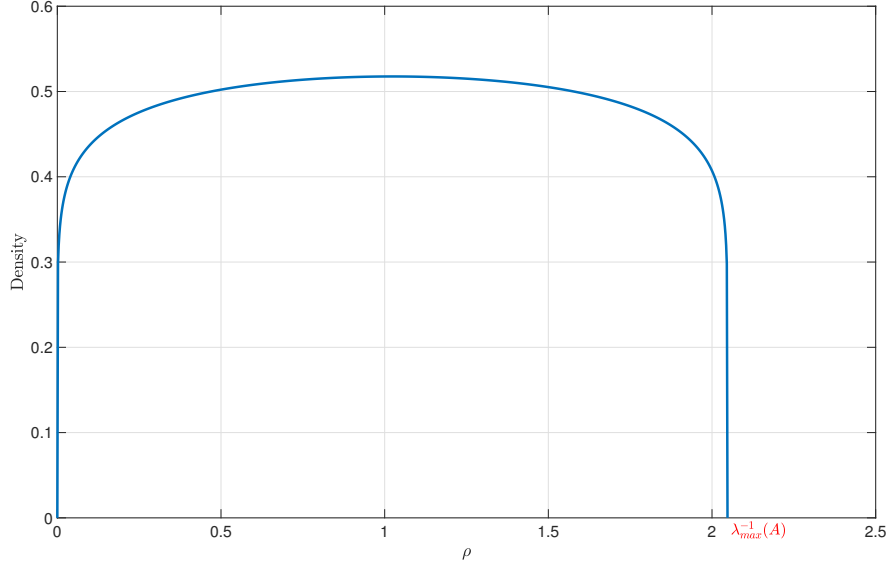


Figure 14: line-plot of a non-standardized $Beta(1.1, 1.1)$ density function, with support from $(0, \lambda_{max}^{-1}(A) = 2.047)$ which we employ as a prior for the spatial parameter ρ^{down} .

have into their estimates.

Furthermore, we assume that all the prior distributions are independent from each other. We use the standard “Metropolis within Gibbs” algorithm, and we obtain an approximation of the posterior densities for each parameter of the model.

We now outline the precise steps of the procedure:

1. **Initialization:** Set up initial values for the parameters: $\beta_{(0)}, \sigma_{(0)}^2, V_{(0)}, \rho_{(0)}^{down}, \rho_{(0)}^{up}$, where $V_{(0)} = \text{diag} \left(v_{1,(0)}^2, \dots, v_{n,(0)}^2 \right)$.
2. **Gibbs Sampling:**
 - a) Draw $\beta_{(1)}$ from the conditional posterior distribution, which is obtained by mixing the likelihood with a normal prior with mean c (a vector of zeros in our simulation) and covariance matrix L . In order to not add any information, we simply set L to be equal to a

diagonal matrix whose entries are infinite (1e12 in our simulation):

$$P(\beta_{(0)}|\mathcal{D}, \sigma_{(0)}^2, V_{(0)}, \rho_{(0)}^{down}, \rho_{(0)}^{up}) = \mathcal{N}(c^*, L^*) \propto \mathcal{L}(\theta|\mathcal{D}) \cdot \mathcal{N}(c, L)$$

$$c^* = \frac{1}{T} \cdot \left(\sum_{t=1}^T X'_t \cdot V_{(0)}^{-1} \cdot X_t + \frac{\sigma_{(0)}^2}{T} \cdot L^{-1} \right)^{-1} \cdot \left(\frac{1}{T} \cdot \sum_{t=1}^T X'_t \cdot V_{(0)}^{-1} \cdot H_t \cdot \Delta y_t + \frac{\sigma_{(0)}^2}{T} \cdot L^{-1} \cdot c \right)$$

$$L^* = \frac{\sigma_{(0)}^2}{T} \cdot \left(\sum_{t=1}^T X'_t \cdot V_{(0)}^{-1} \cdot X_t + \frac{\sigma_{(0)}^2}{T} \cdot L^{-1} \right)^{-1}$$

- b) Draw $\sigma_{(1)}^2$ from the conditional posterior distribution, which is proportional to likelihood times an inverse gamma distribution as a prior:

$$P(\sigma_{(1)}^2|\mathcal{D}, \beta_{(1)}, V_{(0)}, \rho_{(0)}^{down}, \rho_{(0)}^{up}) = \Gamma^{-1}\left(\frac{\theta_1}{2}, \frac{\theta_2}{2}\right) \propto \mathcal{L}(\theta|\mathcal{D}) \cdot \Gamma^{-1}(a, b)$$

$$\theta_1 = nT + 2a \quad \theta_2 = \sum_{t=1}^T \varepsilon'_t \cdot V_{(0)}^{-1} \cdot \varepsilon_t + 2b$$

In practice we draw $\sigma_{(1)}^2$ from θ_2/χ_{θ_1} .

Notice that, setting a and b (the prior's parameters) equal to 0, is like putting a Jefferey's prior on σ^2 . This is exactly what we do.

- c) Draw $v_{i,(1)}$ from the following conditional posterior distribution, proportional to an inverse gamma prior:

$$P(v_{i,(1)}|\mathcal{D}, \sigma_{(1)}^2, \rho_{(0)}^{down}, \rho_{(0)}^{up}) = \Gamma^{-1}\left(\frac{q_1}{2}, \frac{q_2}{2}\right) \propto \mathcal{L}(\theta|\mathcal{D}) \cdot \Gamma^{-1}\left(\frac{r}{2}, \frac{r}{2}\right)$$

$$q_1 = r + T \quad q_2 = \frac{1}{\sigma_{(1)}^2} \cdot \sum_{t=1}^T \varepsilon_{i,t}^2 + r$$

In practice we draw $v_{i,(1)}$ from q_2/χ_{q_1} .

As anticipated above in the paper, since we are confident on the heteroskedastic behavior of industry value added, we set our prior hyperparameter r to be equal to 3 rather than 4, as done in LeSage and Pace (2009).

Replicating this procedure n times, we get a first simulation of matrix $V_{(1)}$.

3. **Metropolis-Hastings:** We now need to draw the spatial coefficients. However we cannot apply a simple Gibbs Sampling, since the conditional posterior distribution is not defined for them. LeSage and Pace (2009)

suggest the adoption of the Metropolis-Hastings algorithm to overcome this problem. To ease notation we set $\rho_1 := \rho^{down}$ and $\rho_2 := \rho^{up}$. The algorithm is the following:

- (a) Draw ρ_1^c (where the c superscript stands for “candidate”) from the (random walk) proposal distribution:

$$\rho_1^c = \rho_{1,(0)} + c_1 \cdot \mathcal{N}(0, 1)$$

- (b) Run a bernoulli experiment to determine the updated value of ρ_1 :

$$\rho_{1,(1)} = \begin{cases} \rho_1^c & \pi \quad (\text{accept}) \\ \rho_{1,(0)} & 1 - \pi \quad (\text{reject}) \end{cases}$$

Where π is equal to $\pi = \min\{1, \psi_{MH_1}\}$ and, setting: $A_\tau(\rho_1) = I_n - \rho_1 \cdot W_1$, we have:

$$\begin{aligned} \psi_{MH_1} = & \frac{|A_\tau(\rho_1^c)|}{|A_\tau(\rho_{1,(0)})|} \cdot \exp \left\{ - \frac{1}{2\sigma_{(1)}^2} \cdot \sum_{t \in t_1}^{T_1} \left[\Delta y'_t \cdot (A_\tau(\rho_1^c)' \cdot V_{(1)}^{-1} \cdot A_\tau(\rho_1^c) - \right. \right. \\ & - A_\tau(\rho_{1,(0)})' \cdot V_{(1)}^{-1} \cdot A_\tau(\rho_{1,(0)}) \cdot \Delta y_t - \\ & \left. \left. - 2\beta' \cdot X'_t \cdot V_{(1)}^{-1} (A_\tau(\rho_1^c) - A_\tau(\rho_{1,(0)})) \cdot \Delta y_t \right] \right\} \cdot \\ & \cdot \left[\frac{(\rho_1^c - 0) \cdot (\lambda_{max}^{-1} - \rho_1^c)}{(\rho_{1,(0)} - 0) \cdot (\lambda_{max}^{-1} - \rho_{1,(0)})} \right]^{d-1} \cdot \mathbf{1}(0 \leq \rho_1^c \leq \lambda_{max}^{-1}) \end{aligned}$$

Basically, we compute the probability to accept the candidate value from the proposal distribution, and then we update the value of ρ_1 by running the bernoulli experiment with such a probability of success. Notice that if we draw a value of ρ_1 outside the support of the beta prior, $\psi_{MH_1} = 0$ and then $\pi = 0$ and we clearly reject the candidate value.

We set d equal to 1.1, on both ρ_1 and ρ_2 ; this is done to resemble a Uniform (0,1) but with less density on its boundary values.

- (c) Once updated ρ_1 , we replicate the procedure for ρ_2 . Setting $A_\gamma(\rho_2) =$

$I_n - \rho_2 \cdot W_2$ we have:

$$\begin{aligned} \psi_{MH_2} = & \frac{|A_\gamma(\rho_2^c)|}{|A_\gamma(\rho_{2,(0)})|} \cdot \exp \left\{ - \frac{1}{2\sigma_{(1)}^2} \cdot \sum_{t \in t_2}^{T_2} \left[\Delta y_t' \cdot (A_\gamma(\rho_2^c)' \cdot V_{(1)}^{-1} \cdot A_\gamma(\rho_2^c) - \right. \right. \\ & - A_\gamma(\rho_{2,(0)})' \cdot V_{(1)}^{-1} \cdot A_\gamma(\rho_{2,(0)}) \cdot \Delta y_t - \\ & \left. \left. - 2\beta' \cdot X_t' \cdot V_{(1)}^{-1} (A_\gamma(\rho_2^c) - A_\gamma(\rho_{2,(0)})) \cdot \Delta y_t \right] \right\} \cdot \\ & \cdot \left[\frac{(\rho_2^c - 0) \cdot (\hat{\lambda}_{max}^{-1} - \rho_2^c)}{(\rho_{2,(0)} - 0) \cdot (\hat{\lambda}_{max}^{-1} - \rho_{2,(0)})} \right]^{d-1} \cdot \mathbf{1}(0 \leq \rho_2^c \leq \hat{\lambda}_{max}^{-1}) \end{aligned}$$

- (d) At this point we need to update the variance of the proposal distributions: if the acceptance rate (number of acceptances over number of iterations of the Markov Chain) of the first parameter ρ_1 falls below 40% we need to reduce the value of c_1 , the so called tuning parameter, which regulates the variance of the proposal distribution. The variance is reduced by rescaling it: $c_1' = \frac{c_1}{1.1}$. In this way, we are able to draw values closer to the current state of ρ_1 , and therefore, we expect to increase the acceptance rate.

On the contrary, if the acceptance rate rises above 60%, we need to increase the tuning parameter, in order to draw values far from the current state, in this way we increase the chance to explore more the low-density parts of the distribution. We increase the variance of the candidate distribution by scaling upward its standard deviation: $c_1' = 1.1 \cdot c_1$.

Clearly we replicate this procedure also for ρ_2 .

4. **Repeat:** Once updated all the values, we replicate steps 2 and 3, 45,000 times to make sure the acceptance rate has converged.
5. **Burn-in:** we drop the first 35,000 iterations of the Markov Chain, thus obtaining a vector of 10,000 observations for each of the parameters, which account for the simulated posterior distributions.

C.4 Simulating the ATE, ADE and ANE

We construct via Monte Carlo the distribution of the ATE, ADE and ANE. In particular we follow these steps:

1. **(Parameters)** Draw ρ^{down} , ρ^{up} , τ and γ from their posterior distributions. To take into account the potential correlation among them, draw from the same iteration of the Bayesian MCMC.

2. **(Style of the plan)** Construct both a TB and an EB simulated fiscal plan, by drawing the style from a distribution which mimics the empirical one.
3. **(Average effects)** Construct ATE, ADE and ANE using the parameters drawn in step 1 and the style drawn in step 2.
4. Repeat 100,000 times steps from 1 though 3, to make sure all the possible combination of styles and parameters are simulated.

Step 2 allows us to claim that the baseline results reported in the paper are robust to different styles of fiscal plans.

Empirical distribution of style of fiscal plans

We are interested in simulating a 2 years fiscal consolidation made of an unexpected part, no announced part and a single year future part to be implemented in the second year of the simulation.

First of all, we want to simulate the unexpected part of the fiscal plan, therefore, we need to look at those years when an unanticipated shock occurs. Define the two sub-samples: $TB^u := \{t : 1, \dots, T \mid tax_t^u > 0\}$ and $EB^u := \{t : 1, \dots, T \mid exp_t^u > 0\}$. Then calculate the mean and the standard deviation of the unexpected component conditional on the occurrence of an unexpected shock:

$$\begin{aligned} \mu_\tau &:= \mathbb{E}(tax_t^u \mid t \in TB^u) & \sigma_\tau &:= \sqrt{\mathbb{V}(tax_t^u \mid t \in TB^u)} \\ \mu_\gamma &:= \mathbb{E}(exp_t^u \mid t \in EB^u) & \sigma_\gamma &:= \sqrt{\mathbb{V}(exp_t^u \mid t \in EB^u)} \end{aligned}$$

In order to simulate a plausible unexpected component of the plan, we draw them from the following distributions:

$$\begin{aligned} \tilde{tax}^u &\sim \mathcal{U}(\mu_\tau - \sigma_\tau, \mu_\tau + \sigma_\tau) \\ \tilde{exp}^u &\sim \mathcal{U}(\mu_\gamma - \sigma_\gamma, \mu_\gamma + \sigma_\gamma) \end{aligned}$$

where the \sim denotes a simulated component.

Concerning the future component, we need to predict what is the value of a one year ahead policy change, conditional on the occurrence of an unexpected policy change. Therefore, we run the following regressions:

$$\begin{aligned} tax_{t,1}^f &= a_\tau + b_\tau \cdot tax_t^u & \text{with: } t \in TB^u \\ exp_{t,1}^f &= a_\gamma + b_\gamma \cdot exp_t^u & \text{with: } t \in EB^u \end{aligned}$$

The estimates of a_τ , b_τ , a_γ , b_γ will be stored and used to predict values of $tax_{t,1}^f$ and $exp_{t,1}^f$, conditional on the occurrence of an unexpected component. At this point we have all the ingredients to outline the steps we do in the construction of a simulated style of the plan:

1. Draw unexpected components from their candidate distributions: $t\tilde{a}x^u \sim \mathcal{U}(\mu_\tau - \sigma_\tau, \mu_\tau + \sigma_\tau)$ and $e\tilde{x}p^u \sim \mathcal{U}(\mu_\gamma - \sigma_\gamma, \mu_\gamma + \sigma_\gamma)$.
2. Predict the future component using the estimates of a_τ , b_τ , a_γ , b_γ . We have: $t\tilde{a}x^f = \hat{a}_\tau + \hat{b}_\tau \cdot t\tilde{a}x^u$ and $e\tilde{x}p^f = \hat{a}_\gamma + \hat{b}_\gamma \cdot e\tilde{x}p^u$.
3. Normalize the value to one: $t\tilde{a}x^u + t\tilde{a}x^f = 1$ and $e\tilde{x}p^u + e\tilde{x}p^f = 1$.

For each iteration of the MC simulation used to approximate the posterior distributions of the ATE, ADE and ANE, we repeat steps 1 through 3 to simulate the style of the plan.

In the first year of the simulation we calculate the effects of TB and EB plans with style given by: $\mathbf{s}_{TB} = [t\tilde{a}x^u \ 0 \ t\tilde{a}x^f]$ and $\mathbf{s}_{EB} = [e\tilde{x}p^u \ 0 \ e\tilde{x}p^f]$ respectively. In the second year of the simulation, the future component of the shock is rolled over and becomes an announced and implemented shock. Therefore we calculate the effects of TB and EB plans with style given by: $\mathbf{s}_{TB} = [0 \ t\tilde{a}x^f \ 0]$ and $\mathbf{s}_{EB} = [0 \ e\tilde{x}p^f \ 0]$ respectively.

D Estimates of Inverted Model

In this section we report the tables of estimates of the inverted model. Firstly, Table VII shows the estimates of Equation (7).

D.1 Model Selection - Vuong Test for Static Spatial Panel Data

We also provide results for a Vuong test of non-nested models, adapted to our spatial specification, as in Wooldridge (2010).

Firstly, the Vuong test (see Vuong (1989)) is meant to discriminate between two misspecified and non-nested models. Basically, we assume there is a hidden true model and we want to choose one of two competing non-nested models which fit the data equally well. The Vuong test calculates and compares the Kullback-Leibler distance between the two and the true model. In practice, is a t-test on the KL divergence. One problem we encounter is that it was developed for one-dimensional iid data, however, we deal with

Table VII: Estimation Results

<i>Inverted Model - Equation (7)</i>												
<i>Parameters</i>	MLE		Bayesian MCMC - Posterior Distributions:									
	$\hat{\theta}_i^{ML}$	MLE Std.	$\mathbb{E}(\theta_i)$	$\sqrt{\mathbb{V}(\theta_i)}$	$Pr(\theta_i < 0)$	5%	10%	16%	50%	84%	90%	95%
ρ^{up} (TB)	0.554	0.103	0.528	0.097	0.000	0.368	0.405	0.432	0.528	0.625	0.653	0.687
τ_u	0.684	1.283	0.815	1.193	0.247	-1.143	-0.712	-0.372	0.814	2.002	2.351	2.778
τ_a	-1.298	0.986	-1.290	0.919	0.920	-2.794	-2.463	-2.202	-1.293	-0.382	-0.112	0.225
τ_f	-0.080	0.426	-0.084	0.391	0.585	-0.726	-0.585	-0.474	-0.082	0.301	0.415	0.562
ρ^{down} (EB)	0.096	0.114	0.125	0.083	0.000	0.014	0.026	0.040	0.112	0.211	0.241	0.281
γ_u	0.073	1.126	0.050	1.034	0.480	-1.650	-1.272	-0.973	0.051	1.073	1.370	1.760
γ_a	1.286	0.617	1.296	0.567	0.011	0.361	0.572	0.732	1.295	1.861	2.023	2.226
γ_f	-0.502	0.282	-0.499	0.259	0.973	-0.923	-0.831	-0.757	-0.499	-0.241	-0.169	-0.075
D2008	-2.984	0.674	-2.934	0.633	1.000	-3.973	-3.744	-3.562	-2.936	-2.307	-2.120	-1.891
D2009	-5.710	0.674	-5.371	0.661	1.000	-6.469	-6.216	-6.025	-5.368	-4.717	-4.529	-4.290

Table VII: θ_i denotes a generic parameter that we estimate. The columns report the following: $\hat{\theta}_i^{ML}$ is the ML point estimate; “MLE Std.” is the standard deviation of the ML estimate, calculated using the analytical Fisher Information Matrix derived in Appendix C.2: $\sqrt{\mathcal{J}(\hat{\theta}_i^{ML})_{ii}^{-1}}$; $\mathbb{E}(\theta_i)$ is the expected value of the posterior distribution; $\sqrt{\mathbb{V}(\theta_i)}$ is the standard deviation of the posterior distribution; $Pr(\theta < 0)$ is the probability that a parameter is negative, calculated by integrating the posterior distribution; p% is the p-th percentile of the posterior distribution. For brevity we don’t report here the Industry Fixed Effects and the Industry specific variances. In the first columns, the spatial parameters also report the type of fiscal plan they are interacted with (in blue).

a panel whose observations are serially uncorrelated but spatially correlated. Wooldridge (2010) shows that the Vuong test can easily be extended to panel data models by accounting for serial correlation in the time series.³⁹ However, in our problem the $n \times 1$ vector of industry observations is iid over time and our asymptotic keeps the cross-sectional dimension, which is spatially correlated, fixed, and then let the time series to go to infinite $T \rightarrow \infty$. Economically speaking this makes sense: we observe those fixed 62 industries over time, however, the cross sectional dimension exceeds the times series one, 37 years. This means that our finite sample distribution will not be a very good approximation of the asymptotic one. However, this is the best we can do, given the data availability.

Let’s derive now the Vuong Test. The quasi-log-likelihood of the baseline model, Equation (6), is:

$$\begin{aligned}
\ell_{t,B}(\underbrace{\rho, \beta, \Omega}_{\theta_B}) &= \log f_B(\Delta y_t | X_t; \theta_B) = -\frac{n}{2} \ln(2\pi) - \frac{1}{2} \cdot \ln(|\Omega|) + \\
&+ \ln(|I_n - \rho^{down} \cdot A \cdot T B_t - \rho^{up} \cdot \hat{A}' \cdot E B_t|) - \frac{1}{2} \cdot \varepsilon_t' \cdot \Omega^{-1} \cdot \varepsilon_t.
\end{aligned}$$

with:

$$\varepsilon_t = \left(I_n - \rho^{down} A T B_t - \rho^{up} \hat{A}' E B_t \right) \cdot \Delta y_t - X_t \cdot \beta.$$

³⁹See Section 13.11.2 - Model Selection Tests.

The sum of the quasi-log-likelihood evaluated at the MLE, $\hat{\theta}_B$, for the baseline model is: $\mathcal{L}_B = \sum_{t=1}^T \ell_{t,B}(\hat{\theta}_B)$. Analogously, for the inverted model, Equation (7), the quasi-log-likelihood is:

$$\begin{aligned} \ell_{t,I}(\underbrace{\tilde{\rho}, \tilde{\beta}, \tilde{\Omega}}_{\theta_I}) &= \log f_I(\Delta y_t | X_t; \theta_I) = -\frac{n}{2} \ln(2\pi) - \frac{1}{2} \cdot \ln(|\tilde{\Omega}|) + \\ &+ \ln(|I_n - \tilde{\rho}^{down} \cdot A \cdot EB_t - \tilde{\rho}^{up} \cdot \hat{A}' \cdot TB_t|) - \frac{1}{2} \cdot \varepsilon_t' \cdot \tilde{\Omega}^{-1} \cdot \varepsilon_t. \end{aligned}$$

with:

$$\varepsilon_t = \left(I_n - \tilde{\rho}^{down} A EB_t - \tilde{\rho}^{up} \hat{A}' TB_t \right) \cdot \Delta y_t - X_t \cdot \tilde{\beta}.$$

The sum of the quasi-log-likelihood evaluated at the MLE, $\hat{\theta}_I$, for the inverted model is: $\mathcal{L}_I = \sum_{t=1}^T \ell_{t,I}(\hat{\theta}_I)$.

Following Wooldridge (2010), let's define the estimator for the variance of the KL divergence as:

$$\hat{\eta}^2 = \frac{1}{T} \cdot \sum_{t=1}^T \left(\ell_{t,B}(\hat{\theta}_B) - \ell_{t,I}(\hat{\theta}_I) \right)^2.$$

Then, the Vuong Model Selection Statistic, VMS, is:

$$\begin{aligned} VMS &= T^{-1/2} \cdot \frac{(\mathcal{L}_B - \mathcal{L}_I)}{\hat{\eta}} \\ &= \frac{\frac{1}{T} \cdot \sum_{t=1}^T \left(\ell_{t,B}(\hat{\theta}_B) - \ell_{t,I}(\hat{\theta}_I) \right)}{\sqrt{\frac{\frac{1}{T} \cdot \sum_{t=1}^T \left(\ell_{t,B}(\hat{\theta}_B) - \ell_{t,I}(\hat{\theta}_I) \right)^2}{T}}} \xrightarrow{d} N(0, 1) \end{aligned}$$

where the standard normal distribution holds under:

$$H_0 : \mathbb{E}[\ell_{t,B}(\theta_B^*)] = \mathbb{E}[\ell_{t,I}(\theta_I^*)]$$

where θ_B^* and θ_I^* are the pseudo-true values of the parameters. Basically, the null hypothesis is saying that the two potentially misspecified models fit the data equally well. Notice that the test is super easy to implement: 1) define the difference: $\hat{d}_t = \ell_{t,B}(\hat{\theta}_B) - \ell_{t,I}(\hat{\theta}_I)$; 2. Regress \hat{d}_t on unity; 3. Run a t-test to verify that the average of the difference is statistically different from zero. We reject the null hypothesis in favor of a better fit to the data of the baseline model if \hat{d}_t is statistically greater than zero. Notice that if this happens it

does not mean that the baseline model is correctly specified (although it could be), however, we can conclude that the baseline model fits better in terms of expected likelihood.

The value we obtain is $VMS = 0.033$ which is clearly not statistically different from zero. Even if positive sign of the statistics points at a better fit of the baseline model against the inverted one, there is not enough statistical evidence to claim that the baseline outperforms on average the inverted model.

D.2 Output Effect of Fiscal Plans in the Inverted Model

We report here the estimated posterior distributions of the ATE, ADE and ANE for fiscal adjustment plans obtained from the estimates of Equation (7) (inverted model).

Table VIII: Average Total, Direct and Network Effects of Fiscal Consolidations in the United States

<i>Inverted Model - Equation (7)</i>											
	$\mathbb{E}(\theta)$	%	$\sqrt{\mathbb{V}(\theta)}$	$Pr(\theta < 0)$	5%	10%	16%	50%	84%	90%	95%
ATE_{TB}	-1.148	1	1.034	0.872	-2.909	-2.481	-2.162	-1.107	-0.131	0.140	0.480
ADE_{TB}	-0.848	0.74	0.756	0.872	-2.106	-1.819	-1.593	-0.835	-0.101	0.107	0.375
ANE_{TB}	-0.300	0.26	0.290	0.872	-0.828	-0.682	-0.572	-0.263	-0.029	0.030	0.102
ATE_{EB}	0.522	1	0.337	0.064	-0.048	0.096	0.203	0.536	0.847	0.936	1.046
ADE_{EB}	0.491	0.94	0.318	0.064	-0.044	0.089	0.188	0.501	0.799	0.886	0.990
ANE_{EB}	0.031	0.06	0.032	0.064	-0.002	0.002	0.005	0.024	0.059	0.073	0.091

Table VIII: descriptive statistics of posterior distributions of Average Effects of a 2 years, 1% magnitude fiscal adjustment plan. 2 years means that results are calculated by cumulating the effect of the first year of the plan and then the second one. The style of the plan is simulated from a distribution which mimics the observed one; see Appendix C.3 for technical details. Columns: $\mathbb{E}(\theta)$ is the expected value of the posterior distribution; % is the share of ATE represented by ADE and ANE. $\sqrt{\mathbb{V}(\theta)}$ is the standard deviations of the posterior distribution; $Pr(\theta < 0)$ is the probability of negative values, calculated by integrating the posterior distribution; “p%” is the p-th percentile of the posterior distribution.

The most important thing to notice is that the ANE of EB plans accounts for only 6% of their ATE, against the 12% of the baseline model. The relevance of ANE of TB plans is basically unaffected, diminishing only by 1% relative to the baseline (from 27% of the ATE to 26%). The statistical significance of the ANE of TB plans declines, since the posterior distribution shrinks towards zero.

E A Potential Theoretical Framework

We show here the theoretical framework which we have in mind when we refer to the theoretical transmission of demand and supply shocks. The model is a

slight modification of Acemoglu, Akcigit, and Kerr (2016), which we adapted to allow for the propagation of a production tax.

The model considers a perfectly competitive economy with n sectors, where the market clearing condition for the generic industry i is:

$$y_i = c_i + \sum_{j=1}^n x_{ji} + G_i \quad (8)$$

where c_i is household's consumption of good produced by industry i ; x_{ij} ⁴⁰ is the quantity of goods produced in industry j used as inputs by industry i ; G_i are government purchases.

$$\sum_{i=1}^n p_i G_i = T + \tau \sum_{i=1}^n p_i y_i \quad (9)$$

Each sector solves the following profit maximization problem:

$$\max_{l_i, \{x_{ij}\}_{j=1}^n} (1 - \tau) \cdot p_i \cdot \underbrace{\left(l_i^{\alpha_i^l} \cdot \left(\prod_{j=1}^n x_{ij}^{\alpha_{ij}} \right)^\rho \right)}_{y_i} - w l_i - \sum_{j=1}^n p_j x_{ij}$$

where τ is a sales/production tax which mimics an excise tax.⁴¹ Notice that the production function is similar to the one in Acemoglu, Carvalho, et al. (2012) and Carvalho (2014). All alpha's are non negative, and we assume constant return to scale: $\alpha_i^l + \rho \cdot \sum_{j=1}^n \alpha_{ij} = 1$. Notice here, that thanks to the Cobb-Douglas specification, ρ can be interpreted as the share of intermediates in production.

The economy is populated by a representative agent, who maximizes utility subject to a budget constraint:

$$\max_{l, \{c_i\}_{i=1}^n} (1 - l)^\lambda \cdot \prod_{i=1}^n c_i^{\beta_i} \quad \text{s.t.} \quad \sum_{i=1}^n p_i c_i \leq w l$$

⁴⁰In Equation (8) we actually have x_{ji} , that is, the amount of good i used as input by industry j ; we then sum over the j -s to obtain the total demand of good i from all the industries.

⁴¹For example, an excise is a special type of sales tax, which is sector-specific. Excise tax might be of two types: ad valorem (percentage of values of a good) and specific (tax paid per unit). The excise tax may be paid by the producer, retailer, and consumer. Moreover, it might be taken on federal, state, and local levels.

with $\sum_{i=1}^n \beta_i = 1$.

Firms and households take all prices as given, and the market-clearing conditions are satisfied in the goods market and the labor market. Government actions are taken as given and the wage is chosen as a numeraire ($w = 1$).

We do not explicitly model a government budget constraints, since during years of fiscal consolidations, spending cuts are not compensated by tax reductions and viceversa. For simplicity we also do not model government debt and deficit.

Households. The household problems returns the following equilibrium conditions:

$$\begin{aligned} \frac{p_i \cdot c_i}{\beta_i} &= \frac{p_j \cdot c_j}{\beta_j} \quad \forall i, j \\ l &= \frac{1}{1 + \lambda} \\ c_i &= \frac{\beta_i}{p_i} \cdot \frac{1}{1 + \lambda} \quad \forall i \\ \sum_{i=1}^n p_i \cdot c_i &= \frac{1}{1 + \lambda} \end{aligned}$$

Therefore, in equilibrium we have:

$$d \log c_i = -d \log p_i \quad \forall i$$

that is, percent changes in consumption of good i only depend on percent changes in the price of the same good (with Cobb-Douglas utility income and substitution effects cancel out).

Firms Firms maximize profits and in equilibrium the following FOCs hold true:

$$\begin{aligned} (1 - \tau) \cdot p_i \cdot \rho \cdot a_{ij} \cdot \frac{y_i}{x_{ij}} &= p_j \\ (1 - \tau) \cdot p_i \cdot \rho \cdot a_i^l \cdot \frac{y_i}{l_i} &= 1 \\ y_i &= l_i^{\alpha_i^l} \cdot \left(\prod_{j=1}^n x_{ij}^{\alpha_{ij}} \right)^\rho \end{aligned}$$

Acemoglu, Akcigit, and Kerr (2016) notes that solving the dual problem (cost minimization) and obtaining the unit cost function is beneficial to the analysis.

The unit cost function is equal to:

$$C(p_1, \dots, p_n) = \underbrace{\left(\frac{1}{a_i^l} \right)^{a_i^l} \cdot \left(\prod_{j=1}^n \left(\frac{1}{\rho \cdot a_{ij}} \right)^{a_{ij}} \right)^\rho}_{:=B_i} \left(\prod_{j=1}^n p_j^{a_{ij}} \right)^\rho.$$

Because of perfect competition, price equals marginal cost. Therefore:

$$(1 - \tau) \cdot p_i = C(p_1, \dots, p_n)$$

By log differentiating the above expression, we have:

$$d \log p_i = \rho \cdot \sum_{j=1}^n a_{ij} \cdot d \log p_j + \frac{\tau}{1 - \tau} d \log \tau$$

The above expression implies that prices are affected only by changes in the production tax τ . Moreover, from profit maximiation we also have:

$$\rho \cdot a_{ij} = \frac{1}{1 - \tau} \cdot \frac{p_j \cdot x_{ij}}{p_i \cdot y_i} \propto \frac{\text{SALES}_{j \rightarrow i}}{\text{SALES}_i}.$$

In other word, if sector i is affected by a tax shock, the effect is propagated downstream to the customers, via x_{ij} . This should be clear if we substitute the firm's FOC condition into the previous expression:

$$d \log p_i = \frac{1}{1 - \tau} \cdot \sum_{j=1}^n \frac{p_j \cdot x_{ij}}{p_i \cdot y_i} \cdot d \log p_j + \frac{\tau}{1 - \tau} d \log \tau.$$

E.1 Network effect of a tax shock

We want to know what is the output effect of a change in the production tax. In order to do so, we need to look at the resource constraint (assuming for

simplicity that $G_i = 0$ for all sectors):

$$y_i = c_i + \sum_{j=1}^n x_{ji}$$

$$\frac{y_i}{c_i} = 1 + \sum_{j=1}^n \frac{x_{ji}}{c_i} \quad \text{plug in: } x_{ji} = (1 - \tau) \cdot p_j \cdot \rho \cdot a_{ji} \cdot \frac{y_j}{p_i} \quad (\text{Firm FOC})$$

$$\frac{y_i}{c_i} = 1 + (1 - \tau) \cdot \rho \sum_{j=1}^n a_{ji} \cdot \frac{p_j \cdot y_j}{p_i \cdot c_i} \quad \text{plug in: } c_i = \frac{\beta_i}{\beta_j} \cdot \frac{p_j \cdot c_j}{p_i} \quad (\text{HH FOC})$$

$$\frac{y_i}{c_i} = 1 + (1 - \tau) \cdot \rho \sum_{j=1}^n a_{ji} \cdot \frac{\beta_i}{\beta_j} \cdot \frac{y_j}{c_j} \quad \text{Denote by: } \theta_i := y_i/c_i$$

$$\theta_i = 1 + (1 - \tau) \cdot \rho \sum_{j=1}^n a_{ji} \cdot \underbrace{\frac{\beta_i}{\beta_j}}_{m_{ij}} \cdot \theta_j$$

Denote by $M := [m_{ij}]_{i,j=1,\dots,n}$. Then, in matrix notation the above expression becomes:

$$\boldsymbol{\theta} = \mathbf{1}_n + (1 - \tau) \cdot \rho \cdot M \cdot \boldsymbol{\theta} \implies \boldsymbol{\theta} = (I_n - (1 - \tau) \cdot \rho \cdot M)^{-1} \cdot \mathbf{1}_n$$

Notice that the equilibrium level of the output-to-consumption ratio, θ_i , has a nice analytical form which, however, depends on τ . Therefore, when τ changes, also this ratio changes and we don't have $d \log y_i = d \log c_i$ as in Acemoglu, Akcigit, and Kerr (2016).

Differentiating the above expression yields:

$$\begin{aligned} d\boldsymbol{\theta} &= \frac{\partial (I_n - (1 - \tau) \cdot \rho \cdot M)^{-1}}{\partial \tau} \cdot \mathbf{1}_n d\tau \\ &= -\rho \cdot (I_n - (1 - \tau) \cdot \rho \cdot M)^{-1} \cdot M \cdot (I_n - (1 - \tau) \cdot \rho \cdot M)^{-1} \cdot \mathbf{1}_n d\tau \end{aligned}$$

Using the $d \log$ notation:

$$d \log \boldsymbol{\theta} = - \underbrace{\tau \cdot \rho \cdot \Theta^{-1} \cdot (I_n - (1 - \tau) \cdot \rho \cdot M)^{-1} \cdot M \cdot (I_n - (1 - \tau) \cdot \rho \cdot M)^{-1} \cdot \mathbf{1}_n}_{:=F} d \log \tau$$

where $\Theta = \text{diag}(\theta_1, \dots, \theta_n)$. Recalling the definition of θ_i , we have:

$$d \log \mathbf{y} = d \log \mathbf{c} - F \cdot \mathbf{1}_n \cdot d \log \tau \implies d \log y_i = d \log c_i - \phi_i \cdot d \log \tau$$

where ϕ_i is the i -th element of vector $F \cdot \mathbf{1}_n$. Notice that if τ were fixed (i.e. $d \log \tau = 0$), percent changes in consumption would be equal to the one of output, as in Acemoglu, Akcigit, and Kerr (2016).

At this point we can find the relationship between output changes and tax shocks. Consider the following three equations we derived earlier:

$$\begin{cases} d \log y_i &= d \log c_i - \phi_i \cdot d \log \tau \\ d \log c_i &= -d \log p_i \\ d \log p_i &= \rho \cdot \sum_{j=1}^n a_{ij} \cdot d \log p_j + \frac{\tau}{1-\tau} d \log \tau \end{cases}$$

Combining the three equations above yields the following expression:

$$\begin{aligned} d \log y_i &= \rho \cdot \sum_{j=1}^n a_{ij} \cdot d \log y_j - \underbrace{\left(\phi_i + \frac{\tau}{1-\tau} - \rho \cdot \sum_{j=1}^n \phi_j \cdot a_{ij} \right)}_{=\psi_i > 0} \cdot d \log \tau \\ &= \rho \cdot \sum_{j=1}^n a_{ij} \cdot d \log y_j - \psi_i \cdot d \log \tau \end{aligned}$$

which is Equation (4) in the paper.

E.2 Network effect of a spending shock

Suppose now that $\tau = 0$ and that the government reduces its purchases from all sectors (i.e. $d \log G_i < 0$). We want to find the relationship between the percent change in output, $d \log y_i$ and percent changes in government purchases $d \log G_i$.

Consider the resource constraint of the economy:

$$\begin{aligned} y_i &= c_i + G_i + \sum_{j=1}^n x_{ji} \quad \text{Log-differentiate} \\ d \log y_i &= \frac{c_i}{y_i} \underbrace{d \log c_i}_{=0 \text{ (} d \log p_i = 0 \text{)}} + \frac{G_i}{y_i} d \log G_i + \sum_{j=1}^n \frac{x_{ji}}{y_i} \cdot d \log x_{ji} \quad (\text{Firm FOC}) \quad x_{ji} = p_j \rho a_{ji} \frac{y_j}{p_i} \\ d \log y_i &= \frac{G_i}{y_i} d \log G_i + \rho \cdot \sum_{j=1}^n a_{ji} \cdot \underbrace{\frac{p_j}{p_i} \frac{y_j}{y_i}}_{:=\hat{a}_{ji}} d \log x_{ji} \\ d \log y_i &= \frac{G_i}{y_i} d \log G_i + \rho \cdot \sum_{j=1}^n \hat{a}_{ji} \cdot d \log x_{ji} \end{aligned}$$

From the firm's FOC, we have:

$$d \log y_i = \underbrace{d \log p_j}_0 + d \log x_{ji} - \underbrace{d \log p_i}_{=0}$$

therefore we can retrieve Equation (2):

$$d \log y_i = \rho \cdot \sum_{j=1}^n \hat{a}_{ji} \cdot d \log y_j + \frac{G_i}{y_i} d \log G_i.$$